

**Paper Specific Instructions**

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.
2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.
4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.
5. In all sections, questions not attempted will result in zero mark. In **Section – A (MCQ)**, wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B (MSQ)**, there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C (NAT)** as well.
6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.
7. The Scribble Pad will be provided for rough work.

**Useful information**

$\mathbb{N}$	set of all natural numbers $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	set of all integers $\{0, \pm 1, \pm 2, \dots\}$
$\mathbb{Q}$	set of all rational numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{C}$	set of all complex numbers
$\mathbb{R}^n$	$n$ -dimensional Euclidean space $\{(x_1, x_2, \dots, x_n) \mid x_j \in \mathbb{R}, 1 \leq j \leq n\}$
$S_n$	group of all permutations of $n$ distinct symbols
$\mathbb{Z}_n$	group of congruence classes of integers modulo $n$
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive $x, y$ and $z$ axes of a three dimensional rectangular coordinate system
$\nabla$	$\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z}$
$M_{m \times n}(\mathbb{R})$	real vector space of all matrices of order $m \times n$ with entries in $\mathbb{R}$
sup	supremum
inf	infimum

**SECTION – A**  
**MULTIPLE CHOICE QUESTIONS (MCQ)**

**Q. 1 – Q.10 carry one mark each.**

Q.1 Which one of the following is TRUE?

- (A)  $\mathbb{Z}_n$  is cyclic if and only if  $n$  is prime
- (B) Every proper subgroup of  $\mathbb{Z}_n$  is cyclic
- (C) Every proper subgroup of  $S_4$  is cyclic
- (D) If every proper subgroup of a group is cyclic, then the group is cyclic

Q.2 Let  $a_n = \frac{b_{n+1}}{b_n}$ , where  $b_1 = 1$ ,  $b_2 = 1$  and  $b_{n+2} = b_n + b_{n+1}$ ,  $n \in \mathbb{N}$ . Then  $\lim_{n \rightarrow \infty} a_n$  is

- (A)  $\frac{1-\sqrt{5}}{2}$       (B)  $\frac{1-\sqrt{3}}{2}$       (C)  $\frac{1+\sqrt{3}}{2}$       (D)  $\frac{1+\sqrt{5}}{2}$

Q.3 If  $\{v_1, v_2, v_3\}$  is a linearly independent set of vectors in a vector space over  $\mathbb{R}$ , then which one of the following sets is also linearly independent?

- (A)  $\{v_1 + v_2 - v_3, 2v_1 + v_2 + 3v_3, 5v_1 + 4v_2\}$
- (B)  $\{v_1 - v_2, v_2 - v_3, v_3 - v_1\}$
- (C)  $\{v_1 + v_2 - v_3, v_2 + v_3 - v_1, v_3 + v_1 - v_2, v_1 + v_2 + v_3\}$
- (D)  $\{v_1 + v_2, v_2 + 2v_3, v_3 + 3v_1\}$

Q.4 Let  $a$  be a positive real number. If  $f$  is a continuous and even function defined on the interval

$[-a, a]$ , then  $\int_{-a}^a \frac{f(x)}{1+e^x} dx$  is equal to

- (A)  $\int_0^a f(x) dx$       (B)  $2 \int_0^a \frac{f(x)}{1+e^x} dx$   
(C)  $2 \int_0^a f(x) dx$       (D)  $2a \int_0^a \frac{f(x)}{1+e^x} dx$

Q.5 The tangent plane to the surface  $z = \sqrt{x^2 + 3y^2}$  at  $(1, 1, 2)$  is given by

- (A)  $x - 3y + z = 0$       (B)  $x + 3y - 2z = 0$   
(C)  $2x + 4y - 3z = 0$       (D)  $3x - 7y + 2z = 0$

- Q.6 In  $\mathbb{R}^3$ , the cosine of the acute angle between the surfaces  $x^2 + y^2 + z^2 - 9 = 0$  and  $z - x^2 - y^2 + 3 = 0$  at the point  $(2, 1, 2)$  is
- (A)  $\frac{8}{5\sqrt{21}}$                       (B)  $\frac{10}{5\sqrt{21}}$                       (C)  $\frac{8}{3\sqrt{21}}$                       (D)  $\frac{10}{3\sqrt{21}}$
- Q.7 Let  $f: \mathbb{R}^3 \rightarrow \mathbb{R}$  be a scalar field,  $\vec{v}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a vector field and let  $\vec{a} \in \mathbb{R}^3$  be a constant vector. If  $\vec{r}$  represents the position vector  $x\hat{i} + y\hat{j} + z\hat{k}$ , then which one of the following is FALSE?
- (A)  $\text{curl}(f \vec{v}) = \text{grad}(f) \times \vec{v} + f \text{curl}(\vec{v})$   
 (B)  $\text{div}(\text{grad}(f)) = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) f$   
 (C)  $\text{curl}(\vec{a} \times \vec{r}) = 2 |\vec{a}| \vec{r}$   
 (D)  $\text{div}\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0$ , for  $\vec{r} \neq \vec{0}$
- Q.8 In  $\mathbb{R}^2$ , the family of trajectories orthogonal to the family of asteroids  $x^{2/3} + y^{2/3} = a^{2/3}$  is given by
- (A)  $x^{4/3} + y^{4/3} = c^{4/3}$                       (B)  $x^{4/3} - y^{4/3} = c^{4/3}$   
 (C)  $x^{5/3} - y^{5/3} = c^{5/3}$                       (D)  $x^{2/3} - y^{2/3} = c^{2/3}$
- Q.9 Consider the vector space  $V$  over  $\mathbb{R}$  of polynomial functions of degree less than or equal to 3 defined on  $\mathbb{R}$ . Let  $T: V \rightarrow V$  be defined by  $(Tf)(x) = f(x) - xf'(x)$ . Then the rank of  $T$  is
- (A) 1                      (B) 2                      (C) 3                      (D) 4
- Q.10 Let  $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$  for  $n \in \mathbb{N}$ . Then which one of the following is TRUE for the sequence  $\{s_n\}_{n=1}^{\infty}$
- (A)  $\{s_n\}_{n=1}^{\infty}$  converges in  $\mathbb{Q}$   
 (B)  $\{s_n\}_{n=1}^{\infty}$  is a Cauchy sequence but does not converge in  $\mathbb{Q}$   
 (C) the subsequence  $\{s_{k^n}\}_{n=1}^{\infty}$  is convergent in  $\mathbb{R}$ , only when  $k$  is even natural number  
 (D)  $\{s_n\}_{n=1}^{\infty}$  is not a Cauchy sequence

**Q. 11 – Q. 30 carry two marks each.**

Q.11 Let  $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n} & , \text{ if } n \text{ is odd} \\ 1 + \frac{1}{2^n} & , \text{ if } n \text{ is even} \end{cases} , n \in \mathbb{N}.$

Then which one of the following is TRUE?

- (A)  $\sup \{a_n \mid n \in \mathbb{N}\} = 3$  and  $\inf \{a_n \mid n \in \mathbb{N}\} = 1$
- (B)  $\liminf (a_n) = \limsup (a_n) = \frac{3}{2}$
- (C)  $\sup \{a_n \mid n \in \mathbb{N}\} = 2$  and  $\inf \{a_n \mid n \in \mathbb{N}\} = 1$
- (D)  $\liminf (a_n) = 1$  and  $\limsup (a_n) = 3$

Q.12 Let  $a, b, c \in \mathbb{R}$ . Which of the following values of  $a, b, c$  do NOT result in the convergence of the series

$$\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c} \quad ?$$

- (A)  $|a| < 1, b \in \mathbb{R}, c \in \mathbb{R}$
- (B)  $a = 1, b > 1, c \in \mathbb{R}$
- (C)  $a = 1, b \geq 0, c < 1$
- (D)  $a = -1, b \geq 0, c > 0$

Q.13 Let  $a_n = n + \frac{1}{n}, n \in \mathbb{N}$ . Then the sum of the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_{n+1}}{n!}$  is

- (A)  $e^{-1} - 1$
- (B)  $e^{-1}$
- (C)  $1 - e^{-1}$
- (D)  $1 + e^{-1}$

Q.14 Let  $a_n = \frac{(-1)^n}{\sqrt{1+n}}$  and let  $c_n = \sum_{k=0}^n a_{n-k} a_k$ , where  $n \in \mathbb{N} \cup \{0\}$ . Then which one of the following is TRUE?

- (A) Both  $\sum_{n=0}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} c_n$  are convergent
- (B)  $\sum_{n=0}^{\infty} a_n$  is convergent but  $\sum_{n=1}^{\infty} c_n$  is not convergent
- (C)  $\sum_{n=1}^{\infty} c_n$  is convergent but  $\sum_{n=0}^{\infty} a_n$  is not convergent
- (D) Neither  $\sum_{n=0}^{\infty} a_n$  nor  $\sum_{n=1}^{\infty} c_n$  is convergent

Q.15 Suppose that  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  are differentiable functions such that  $f$  is strictly increasing and  $g$  is strictly decreasing. Define  $p(x) = f(g(x))$  and  $q(x) = g(f(x))$ ,  $\forall x \in \mathbb{R}$ . Then, for  $t > 0$ , the sign of  $\int_0^t p'(x) (q'(x) - 3) dx$  is

- (A) positive                      (B) negative                      (C) dependent on  $t$                       (D) dependent on  $f$  and  $g$

Q.16 For  $x \in \mathbb{R}$ , let  $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ . Then which one of the following is FALSE?

- (A)  $\lim_{x \rightarrow 0} \frac{f(x)}{x} = 0$   
 (B)  $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 0$   
 (C)  $\frac{f(x)}{x^2}$  has infinitely many maxima and minima on the interval  $(0,1)$   
 (D)  $\frac{f(x)}{x^4}$  is continuous at  $x = 0$  but not differentiable at  $x = 0$

Q.17 Let  $f(x, y) = \begin{cases} \frac{xy}{(x^2+y^2)^\alpha}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$

Then which one of the following is TRUE for  $f$  at the point  $(0,0)$ ?

- (A) For  $\alpha = 1$ ,  $f$  is continuous but not differentiable  
 (B) For  $\alpha = \frac{1}{2}$ ,  $f$  is continuous and differentiable  
 (C) For  $\alpha = \frac{1}{4}$ ,  $f$  is continuous and differentiable  
 (D) For  $\alpha = \frac{3}{4}$ ,  $f$  is neither continuous nor differentiable

Q.18 Let  $a, b \in \mathbb{R}$  and let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a thrice differentiable function. If  $z = e^u f(v)$ , where  $u = ax + by$  and  $v = ax - by$ , then which one of the following is TRUE?

- (A)  $b^2 z_{xx} - a^2 z_{yy} = 4a^2 b^2 e^u f'(v)$                       (B)  $b^2 z_{xx} - a^2 z_{yy} = -4e^u f'(v)$   
 (C)  $bz_x + az_y = abz$                       (D)  $bz_x + az_y = -abz$

Q.19 Consider the region  $D$  in the  $yz$  plane bounded by the line  $y = \frac{1}{2}$  and the curve  $y^2 + z^2 = 1$ , where  $y \geq 0$ . If the region  $D$  is revolved about the  $z$ -axis in  $\mathbb{R}^3$ , then the volume of the resulting solid is

- (A)  $\frac{\pi}{\sqrt{3}}$                       (B)  $\frac{2\pi}{\sqrt{3}}$                       (C)  $\frac{\pi\sqrt{3}}{2}$                       (D)  $\pi\sqrt{3}$

Q.20 If  $\vec{F}(x, y) = (3x - 8y)\hat{i} + (4y - 6xy)\hat{j}$  for  $(x, y) \in \mathbb{R}^2$ , then  $\oint_C \vec{F} \cdot d\vec{r}$ , where  $C$  is the boundary of the triangular region bounded by the lines  $x = 0$ ,  $y = 0$  and  $x + y = 1$  oriented in the anti-clockwise direction, is

- (A)  $\frac{5}{2}$                       (B) 3                      (C) 4                      (D) 5

Q.21 Let  $U, V$  and  $W$  be finite dimensional real vector spaces,  $T: U \rightarrow V$ ,  $S: V \rightarrow W$  and  $P: W \rightarrow U$  be linear transformations. If  $\text{range}(ST) = \text{nullspace}(P)$ ,  $\text{nullspace}(ST) = \text{range}(P)$  and  $\text{rank}(T) = \text{rank}(S)$ , then which one of the following is TRUE?

- (A) nullity of  $T = \text{nullity of } S$   
 (B) dimension of  $U \neq \text{dimension of } W$   
 (C) If dimension of  $V = 3$ , dimension of  $U = 4$ , then  $P$  is not identically zero  
 (D) If dimension of  $V = 4$ , dimension of  $U = 3$  and  $T$  is one-one, then  $P$  is identically zero

Q.22 Let  $y(x)$  be the solution of the differential equation  $\frac{dy}{dx} + y = f(x)$ , for  $x \geq 0$ ,  $y(0) = 0$ , where

$$f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}. \quad \text{Then } y(x) =$$

- (A)  $2(1 - e^{-x})$  when  $0 \leq x < 1$  and  $2(e - 1)e^{-x}$  when  $x \geq 1$   
 (B)  $2(1 - e^{-x})$  when  $0 \leq x < 1$  and 0 when  $x \geq 1$   
 (C)  $2(1 - e^{-x})$  when  $0 \leq x < 1$  and  $2(1 - e^{-1})e^{-x}$  when  $x \geq 1$   
 (D)  $2(1 - e^{-x})$  when  $0 \leq x < 1$  and  $2e^{1-x}$  when  $x \geq 1$

Q.23 An integrating factor of the differential equation  $(y + \frac{1}{3}y^3 + \frac{1}{2}x^2)dx + \frac{1}{4}(x + xy^2)dy = 0$  is

- (A)  $x^2$                       (B)  $3 \log_e x$                       (C)  $x^3$                       (D)  $2 \log_e x$

Q.24 A particular integral of the differential equation  $y'' + 3y' + 2y = e^{e^x}$  is

- (A)  $e^{e^x}e^{-x}$                       (B)  $e^{e^x}e^{-2x}$                       (C)  $e^{e^x}e^{2x}$                       (D)  $e^{e^x}e^x$

Q.25 Let  $G$  be a group satisfying the property that  $f: G \rightarrow \mathbb{Z}_{221}$  is a homomorphism implies  $f(g) = 0, \forall g \in G$ . Then a possible group  $G$  is

- (A)  $\mathbb{Z}_{21}$                       (B)  $\mathbb{Z}_{51}$                       (C)  $\mathbb{Z}_{91}$                       (D)  $\mathbb{Z}_{119}$

Q.26 Let  $H$  be the quotient group  $\mathbb{Q}/\mathbb{Z}$ . Consider the following statements.

- I. Every cyclic subgroup of  $H$  is finite.
- II. Every finite cyclic group is isomorphic to a subgroup of  $H$ .

Which one of the following holds?

- (A) I is TRUE but II is FALSE
- (B) II is TRUE but I is FALSE
- (C) both I and II are TRUE
- (D) neither I nor II is TRUE

Q.27 Let  $I$  denote the  $4 \times 4$  identity matrix. If the roots of the characteristic polynomial of a  $4 \times 4$  matrix

$M$  are  $\pm\sqrt{\frac{1\pm\sqrt{5}}{2}}$ , then  $M^8 =$

- (A)  $I + M^2$
- (B)  $2I + M^2$
- (C)  $2I + 3M^2$
- (D)  $3I + 2M^2$

Q.28 Consider the group  $\mathbb{Z}^2 = \{(a, b) \mid a, b \in \mathbb{Z}\}$  under component-wise addition. Then which of the following is a subgroup of  $\mathbb{Z}^2$  ?

- (A)  $\{(a, b) \in \mathbb{Z}^2 \mid ab = 0\}$
- (B)  $\{(a, b) \in \mathbb{Z}^2 \mid 3a + 2b = 15\}$
- (C)  $\{(a, b) \in \mathbb{Z}^2 \mid 7 \text{ divides } ab\}$
- (D)  $\{(a, b) \in \mathbb{Z}^2 \mid 2 \text{ divides } a \text{ and } 3 \text{ divides } b\}$

Q.29 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function and let  $J$  be a bounded open interval in  $\mathbb{R}$ . Define

$$W(f, J) = \sup \{f(x) \mid x \in J\} - \inf \{f(x) \mid x \in J\} .$$

Which one of the following is FALSE?

- (A)  $W(f, J_1) \leq W(f, J_2)$  if  $J_1 \subset J_2$
- (B) If  $f$  is a bounded function in  $J$  and  $J \supset J_1 \supset J_2 \cdots \supset J_n \supset \cdots$  such that the length of the interval  $J_n$  tends to 0 as  $n \rightarrow \infty$ , then  $\lim_{n \rightarrow \infty} W(f, J_n) = 0$
- (C) If  $f$  is discontinuous at a point  $a \in J$ , then  $W(f, J) \neq 0$
- (D) If  $f$  is continuous at a point  $a \in J$ , then for any given  $\epsilon > 0$  there exists an interval  $I \subset J$  such that  $W(f, I) < \epsilon$

Q.30 For  $x > \frac{-1}{2}$ , let  $f_1(x) = \frac{2x}{1+2x}$ ,  $f_2(x) = \log_e(1+2x)$  and  $f_3(x) = 2x$ . Then which one of the following is TRUE?

- (A)  $f_3(x) < f_2(x) < f_1(x)$  for  $0 < x < \frac{\sqrt{3}}{2}$   
 (B)  $f_1(x) < f_3(x) < f_2(x)$  for  $x > 0$   
 (C)  $f_1(x) + f_2(x) < \frac{f_3(x)}{2}$  for  $x > \frac{\sqrt{3}}{2}$   
 (D)  $f_2(x) < f_1(x) < f_3(x)$  for  $x > 0$

### SECTION - B

#### MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let  $f: \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$  be defined by  $f(x) = x + \frac{1}{x^3}$ . On which of the following interval(s) is  $f$  one-one?

- (A)  $(-\infty, -1)$       (B)  $(0, 1)$       (C)  $(0, 2)$       (D)  $(0, \infty)$

Q.32 The solution(s) of the differential equation  $\frac{dy}{dx} = (\sin 2x) y^{1/3}$  satisfying  $y(0) = 0$  is (are)

- (A)  $y(x) = 0$       (B)  $y(x) = -\sqrt{\frac{8}{27}} \sin^3 x$   
 (C)  $y(x) = \sqrt{\frac{8}{27}} \sin^3 x$       (D)  $y(x) = \sqrt{\frac{8}{27}} \cos^3 x$

Q.33 Suppose  $f, g, h$  are permutations of the set  $\{\alpha, \beta, \gamma, \delta\}$ , where

$f$  interchanges  $\alpha$  and  $\beta$  but fixes  $\gamma$  and  $\delta$ ,

$g$  interchanges  $\beta$  and  $\gamma$  but fixes  $\alpha$  and  $\delta$ ,

$h$  interchanges  $\gamma$  and  $\delta$  but fixes  $\alpha$  and  $\beta$ .

Which of the following permutations interchange(s)  $\alpha$  and  $\delta$  but fix(es)  $\beta$  and  $\gamma$ ?

- (A)  $f \circ g \circ h \circ g \circ f$     (B)  $g \circ h \circ f \circ h \circ g$     (C)  $g \circ f \circ h \circ f \circ g$     (D)  $h \circ g \circ f \circ g \circ h$

Q.34 Let  $P$  and  $Q$  be two non-empty disjoint subsets of  $\mathbb{R}$ . Which of the following is (are) FALSE?

- (A) If  $P$  and  $Q$  are compact, then  $P \cup Q$  is also compact  
 (B) If  $P$  and  $Q$  are not connected, then  $P \cup Q$  is also not connected  
 (C) If  $P \cup Q$  and  $P$  are closed, then  $Q$  is closed  
 (D) If  $P \cup Q$  and  $P$  are open, then  $Q$  is open



Q.35 Let  $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$  denote the group of non-zero complex numbers under multiplication. Suppose  $Y_n = \{z \in \mathbb{C} \mid z^n = 1\}$ ,  $n \in \mathbb{N}$ . Which of the following is (are) subgroup(s) of  $\mathbb{C}^*$ ?

- (A)  $\bigcup_{n=1}^{100} Y_n$       (B)  $\bigcup_{n=1}^{\infty} Y_{2^n}$       (C)  $\bigcup_{n=100}^{\infty} Y_n$       (D)  $\bigcup_{n=1}^{\infty} Y_n$

Q.36 Suppose  $\alpha, \beta, \gamma \in \mathbb{R}$ . Consider the following system of linear equations.

$x + y + z = \alpha$ ,  $x + \beta y + z = \gamma$ ,  $x + y + \alpha z = \beta$ . If this system has at least one solution, then which of the following statements is (are) TRUE?

- (A) If  $\alpha = 1$  then  $\gamma = 1$       (B) If  $\beta = 1$  then  $\gamma = \alpha$   
 (C) If  $\beta \neq 1$  then  $\alpha = 1$       (D) If  $\gamma = 1$  then  $\alpha = 1$

Q.37 Let  $m, n \in \mathbb{N}$ ,  $m < n$ ,  $P \in M_{n \times m}(\mathbb{R})$ ,  $Q \in M_{m \times n}(\mathbb{R})$ . Then which of the following is (are) NOT possible?

- (A)  $\text{rank}(PQ) = n$   
 (B)  $\text{rank}(QP) = m$   
 (C)  $\text{rank}(PQ) = m$   
 (D)  $\text{rank}(QP) = \left\lceil \frac{m+n}{2} \right\rceil$ , the smallest integer larger than or equal to  $\frac{m+n}{2}$

Q.38 If  $\vec{F}(x, y, z) = (2x + 3yz)\hat{i} + (3xz + 2y)\hat{j} + (3xy + 2z)\hat{k}$  for  $(x, y, z) \in \mathbb{R}^3$ , then which among the following is (are) TRUE?

- (A)  $\nabla \times \vec{F} = \vec{0}$   
 (B)  $\oint_C \vec{F} \cdot d\vec{r} = 0$  along any simple closed curve  $C$   
 (C) There exists a scalar function  $\phi: \mathbb{R}^3 \rightarrow \mathbb{R}$  such that  $\nabla \cdot \vec{F} = \phi_{xx} + \phi_{yy} + \phi_{zz}$   
 (D)  $\nabla \cdot \vec{F} = 0$

Q.39 Which of the following subsets of  $\mathbb{R}$  is (are) connected?

- (A)  $\{x \in \mathbb{R} \mid x^2 + x > 4\}$       (B)  $\{x \in \mathbb{R} \mid x^2 + x < 4\}$   
 (C)  $\{x \in \mathbb{R} \mid |x| < |x - 4|\}$       (D)  $\{x \in \mathbb{R} \mid |x| > |x - 4|\}$

- Q.40 Let  $S$  be a subset of  $\mathbb{R}$  such that 2018 is an interior point of  $S$ . Which of the following is (are) TRUE?
- (A)  $S$  contains an interval
- (B) There is a sequence in  $S$  which does not converge to 2018
- (C) There is an element  $y \in S$ ,  $y \neq 2018$  such that  $y$  is also an interior point of  $S$
- (D) There is a point  $z \in S$ , such that  $|z - 2018| = 0.002018$

### SECTION – C

#### NUMERICAL ANSWER TYPE (NAT)

**Q. 41 – Q. 50 carry one mark each.**

Q.41 The order of the element  $(1\ 2\ 3)(2\ 4\ 5)(4\ 5\ 6)$  in the group  $S_6$  is \_\_\_\_\_

Q.42 Let  $\phi(x, y, z) = 3y^2 + 3yz$  for  $(x, y, z) \in \mathbb{R}^3$ . Then the absolute value of the directional derivative of  $\phi$  in the direction of the line  $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-2}$ , at the point  $(1, -2, 1)$  is \_\_\_\_\_

Q.43 Let  $f(x) = \sum_{n=0}^{\infty} (-1)^n x(x-1)^n$  for  $0 < x < 2$ . Then the value of  $f\left(\frac{\pi}{4}\right)$  is \_\_\_\_\_

Q.44 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be given by

$$f(x, y) = \begin{cases} \frac{x^2 y (x - y)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then  $\frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) - \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right)$  at the point  $(0, 0)$  is \_\_\_\_\_

Q.45 Let  $f(x, y) = \sqrt{x^3 y} \sin\left(\frac{\pi}{2} e^{\left(\frac{y}{x}-1\right)}\right) + xy \cos\left(\frac{\pi}{3} e^{\left(\frac{x}{y}-1\right)}\right)$  for  $(x, y) \in \mathbb{R}^2$ ,  $x > 0$ ,  $y > 0$ .

Then  $f_x(1, 1) + f_y(1, 1) =$  \_\_\_\_\_

Q.46 Let  $f: [0, \infty) \rightarrow [0, \infty)$  be continuous on  $[0, \infty)$  and differentiable on  $(0, \infty)$ . If

$$f(x) = \int_0^x \sqrt{f(t)} dt, \text{ then } f(6) = \text{_____}$$

Q.47 Let  $a_n = \frac{(1+(-1)^n)}{2^n} + \frac{(1+(-1)^{n-1})}{3^n}$ . Then the radius of convergence of the power series  $\sum_{n=1}^{\infty} a_n x^n$  about  $x = 0$  is \_\_\_\_\_

Q.48 Let  $A_6$  be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in  $A_6$  is \_\_\_\_\_

Q.49 Let  $W_1$  be the real vector space of all  $5 \times 2$  matrices such that the sum of the entries in each row is zero. Let  $W_2$  be the real vector space of all  $5 \times 2$  matrices such that the sum of the entries in each column is zero. Then the dimension of the space  $W_1 \cap W_2$  is \_\_\_\_\_

Q.50 The coefficient of  $x^4$  in the power series expansion of  $e^{\sin x}$  about  $x = 0$  is \_\_\_\_\_ (correct up to three decimal places).

**Q. 51 – Q. 60 carry two marks each.**

Q.51 Let  $a_k = (-1)^{k-1}$ ,  $s_n = a_1 + a_2 + \dots + a_n$  and  $\sigma_n = (s_1 + s_2 + \dots + s_n)/n$ , where  $k, n \in \mathbb{N}$ . Then  $\lim_{n \rightarrow \infty} \sigma_n$  is \_\_\_\_\_ (correct up to one decimal place).

Q.52 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be such that  $f''$  is continuous on  $\mathbb{R}$  and  $f(0) = 1$ ,  $f'(0) = 0$  and  $f''(0) = -1$ .

Then  $\lim_{x \rightarrow \infty} \left( f \left( \sqrt{\frac{2}{x}} \right) \right)^x$  is \_\_\_\_\_ (correct up to three decimal places).

Q.53 Suppose  $x, y, z$  are positive real numbers such that  $x + 2y + 3z = 1$ . If  $M$  is the maximum value of  $xyz^2$ , then the value of  $\frac{1}{M}$  is \_\_\_\_\_

Q.54 If the volume of the solid in  $\mathbb{R}^3$  bounded by the surfaces

$$x = -1, \quad x = 1, \quad y = -1, \quad y = 1, \quad z = 2, \quad y^2 + z^2 = 2$$

is  $\alpha - \pi$ , then  $\alpha =$  \_\_\_\_\_

Q.55 If  $\alpha = \int_{\pi/6}^{\pi/3} \frac{\sin t + \cos t}{\sqrt{\sin 2t}} dt$ , then the value of  $\left(2 \sin \frac{\alpha}{2} + 1\right)^2$  is \_\_\_\_\_

Q.56 The value of the integral

$$\int_0^1 \int_x^1 y^4 e^{xy^2} dy dx$$

is \_\_\_\_\_ (correct up to three decimal places).

Q.57 Suppose  $Q \in M_{3 \times 3}(\mathbb{R})$  is a matrix of rank 2. Let  $T: M_{3 \times 3}(\mathbb{R}) \rightarrow M_{3 \times 3}(\mathbb{R})$  be the linear transformation defined by  $T(P) = QP$ . Then the rank of  $T$  is \_\_\_\_\_

Q.58 The area of the parametrized surface

$$S = \{(2 + \cos u) \cos v, (2 + \cos u) \sin v, \sin u\} \in \mathbb{R}^3 \mid 0 \leq u \leq \frac{\pi}{2}, 0 \leq v \leq \frac{\pi}{2}\}$$

is \_\_\_\_\_ (correct up to two decimal places).

Q.59 If  $x(t)$  is the solution to the differential equation  $\frac{dx}{dt} = x^2 t^3 + xt$ , for  $t > 0$ , satisfying  $x(0) = 1$ , then the value of  $x(\sqrt{2})$  is \_\_\_\_\_ (correct up to two decimal places).

Q.60 If  $y(x) = v(x) \sec x$  is the solution of  $y'' - (2 \tan x) y' + 5y = 0$ ,  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ , satisfying  $y(0) = 0$  and  $y'(0) = \sqrt{6}$ , then  $v\left(\frac{\pi}{6\sqrt{6}}\right)$  is \_\_\_\_\_ (correct up to two decimal places).

**END OF THE QUESTION PAPER**