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IIT JAM-MA MATHEMATICS (FMTP)

Attempt ALL the 60 questions.

There are a total of 60 questions carrying 100 marks.

Section-A contains a total of 30 **Multiple Choice Questions (MCQ).** Q.1 - Q.10 carry 1 mark each and Questi ons Q.11 - Q.30 carry 2 marks each.

Section-B contains a total of 10 **Multiple Select Questions (MSQ).** Questions Q.31 - Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

Section-C contains a total of 20 **Numerical Answer Type (NAT)** questions. Questions Q.41 - Q.60 belong to this section and carry a total of 30 marks. Q.41 - Q.50 carry 1 mark each and Questions Q.51 - Q.60 carry 2 marks each.

In **Section-A** for all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section-B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section-C** (NAT) as well.

Time : 3 Hours MAX.MARKS : 100 MARKS SCORED : ☐

SECTION-A (Q. 1-30): MULTIPLE CHOICE QUESTIONs (MCQs)

1. Let $a_n = \frac{n+1}{n^2+1^2} + \frac{n+2}{n^2 2^2} + ... + \frac{1}{n}$ for $n \in \mathbb{N}$, then which one of the following is true for the sequence $\{a_n\}$.

(A) $\{a_n\}$ be a Cauchy sequence and Converges in Q

(B) $\{a_n\}$ not a Cauchy sequence .

(C) $\{a_n\}$ be a Cauchy sequence and Converges in Q^c.

(D) only one sub sequence of $\{a_n\}$ exist which converging on Q.

2. Unit normal vector to the level surface $x^2y + 2xz = 4$ at the point (2, -2,3) is,

(A) $\frac{i-2j+2k}{3}$ (B) $\frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}k$

(C) $-\frac{i}{3}+\frac{2}{3}j-\frac{2}{3}k$ (D) $\frac{1}{3}i+\frac{2}{3}j-\frac{2}{3}k$

3. The value of integral $\int_0^{\pi}\frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx$ is

(A) $\frac{\pi}{2}$ (B) π (C) 2π (D) $\frac{3\pi}{2}$

4. The Cartesian equation of the curve whose gradient at (x,y) is $\frac{y^3}{e^{2x}+y^2}$ and passes through the point (0,1) is,

(A) $e^{-2x}=\frac{1}{y^2}\log\frac{1}{y^2}$ (B) $e^{-2x}y^2=\log y^2+1$

(C) $e^{-2x}=\frac{1}{y^2}\log\frac{e}{y^2}$ (D) None of these

5. If f(x) = x^3 + $4x^2$ + $\lambda x+1$ is a monotonically function of x in the largest possible interval $\left(-2,-\frac{2}{3}\right)$ Then

(A) $\lambda=4$ (B) $\lambda=2$ (C) $\lambda=-1$ (D) λ has no real value

6. Consider the vector space V over **R** of polynomials functions of degree less than or equal to 3 defined on **R.** Let $T:V\rightarrow V$ be a linear transformation, defined by (Tf) (x) = f(x) + f'(x) - f''(x). Then rank T is

(A) 1 (B) 2 (C) 3 (D) 4

7. Let $S=\left\{\begin{bmatrix} a & a \\ a & a\end{bmatrix}\Big/ a\in\mathbb{R}-\{0\}\right\}$ is a group with respect to matrix multiplication. Then inverse of $A=\begin{bmatrix} 3 & 3 \\ 3 & 3\end{bmatrix}$ is,

(A) $A' = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(B) $A' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\Rightarrow A' = \begin{bmatrix} 1/12 & 1/12 \\ 1/12 & 1/12 \end{bmatrix}$

(D) $A' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

8. Which of the following series is not convergent.

(A) $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + ...$

(B) $\sum \frac{n+1}{n^P}, p > q$

(C) $\sum \left(\sqrt{n^4+1} - \sqrt{n^4-1} \right)$

(D) None of these

9. The value of K for which the set of equations x + ky + 3z = 0, 3x + ky - 2z = 0, 2x + 3y - 4z = 0, has a non-trivial solution over the set of rationales is

(A) 15 (B) $\frac{31}{2}$ (C) 16 (D) $\frac{33}{2}$

10. $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} e^{\frac{r}{n}}$ is

(A) e + 1 (B) e - 1 (C) 1 - e (D) e

11. Which of the following statements is true for the function

$$f(x) = \begin{cases} \sqrt{x} & , \quad x \geq 1 \\ x^3 & , \quad 0 \leq x < 1 \\ \frac{x^3}{3} - 4x & , \quad 0 < 0 \end{cases}$$

(A) It is monotonic increasing $\forall x \in \mathbb{R}$

(B) f'(x) fails to exist for three distinct real values of x

(C) f'(x) changes its sign twice as x varies from $-\infty$ to $\frac{1}{2}$

(D) The function attains its extreme values at x, and and x2 such that $x_1, x_2 > 0$

12. The value of integral $\int_0^{\pi} \frac{\sin(2x).\sin\left(\frac{\pi}{2}\cos x\right)dx}{2x-\pi}$, is

(A) $\frac{1}{\pi^2}$ (B) $\frac{2}{\pi^2}$ (C) $\frac{4}{\pi^2}$ (D) $\frac{8}{\pi^2}$

13. Let A be a 3 x 4 matrix of rank 2. Then the rank of A^TA, where A^T denotes the transpose of A. is,

(A) exactly 2 (B) exactly 1

(C) at most 3, and atleast 2 (D) at most 2, but not necessarily 2

14. Let $f:\mathbb{R}\to\mathbb{R}$ be a twice continuously differentiable function, with, f(0) = f(1) = f'(0) = 0, Then

(A) f'' is the zero function (B) f'' (0) is zero

(C) f''(x) = 0 for some $x\in[0,1]$ (D) f'' never vanishes

15. $\lim_{n\to\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{2}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{6}}+..........+\frac{1}{\sqrt{2n}+\sqrt{2n+2}}\right)$ **is**

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$ (D) $\frac{1}{\sqrt{2}+1}$

16. Let $y_1(x) = e^{-x}$ and $y_2(x) = (x^2 - 2x + 2)$ are two linearly independent solutions of DE. xy'' + (x - 2) y' - 2y = 0. If $y(x) = C_1e^{-x} + C_2(x^2 - 2x + 2) + V(x)$ be the general Sol^n of DE xy'' + (x - 2) y' - 2y = x^3, then V(0) is equal to ?

(A) -4 (B) 6 (C) -6 (D) 2

17. Let G = {f_1. f_2, f_3, f_4} with respect to composite mapping be a group, where f_1 = x, f_2 = -x, $f_3=\frac{1}{x}$ and $f_4=\frac{1}{4}$

(A) f_1 (B) f_2 (C) f_4 (D) self invertible

18. The number of Cosets of H in G, Where $G=(\mathbb{Z},+)$ and $H=(4\mathbb{Z},+)$ is

(A) 0 (B) 1 (C) 2 (D) 4

19. Let $S=\{(x,y,z)\in\mathbb{R}^3/x^2+y^2+z^2-4x+2z=0, x\geq 0\}$ be a given surface and $\vec{F}=(y^2+xz^2+x^2)i+(z^2+yx^2+y^2)j+(x^2+zy^2+z^2)k$ be a vector field defined on S. Then the value of integral $\iint_S(\nabla x\vec{F}).\hat{n}ds$, where $\hat{n}$ is outward unit normal to S, is given by,

(A) 2π (B) 3π (C) 4π (D) 0

20. Let S be the surface of cylinder $x^2 + y^2 = 1$ Bounded by the sphere $x^2 + y^2 + z^2 = 4$. Let $\vec{F}=xy\,i+yz^2j+z\,xk$ be a vector field defined on S and $\hat{n}$ is outward unit normal to S,

Then the value of $\iint_S\vec{F}.\hat{n}ds$ is,

(A) $\sqrt{3}\,\pi$ (B) $2\sqrt{3}\,\pi$ (C) 2π (D) 0

21. The area bounded by the curves
$y = -x^2 + 6x - 5$, $y = -x^2 + 4x - 3$ and the straight line $y = 3x - 15$ is given by

(A) $\frac{63}{6}$ (B) $\frac{53}{3}$ (C) $\frac{73}{6}$ (D) $\frac{83}{3}$

22. Using the fact that $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=\log 2$, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)}$ is equal to,

(A) 1 - 2 log2 (B) 1 + log 2 (C) $(\log 2)^2$ (D) $-(\log)^2$

23. Let V be the real vector space of real polynomials of degree <3 and $T : V\rightarrow V$ be the L . T. defined by P(t) to Q(t) where Q(t) = P(t + b). Then the matrix of T w.r. to basis 1, t, t^2 of V is,

(A) $\begin{bmatrix} 1 & b & b^2 \\ 0 & a & 2ab \\ 0 & 0 & a^2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & a & a^2 \\ 0 & b & 2ab \\ 0 & 0 & b^2 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & b & b^2 \\ a & a & 0 \\ 0 & b & a^2 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & a & a^2 \\ b & b & 0 \\ 0 & a & b^2 \end{bmatrix}$

24. Which of the following subsets of $\mathbb{R}^4$ is a Basis of $\mathbb{R}^4$?

B_1 = {(1,0,0,0), (1,1,0,0), (1,1,1,1)}

B_2 = {(1,0,0,0), (1,2,0,0), (1,2,3,0) (1,2,3,4)}

B_3 = {(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5, 5, 0, 0)}

(A) B_1 and B_2 but not B_3

(B) B_1, B_2 but not B_2

(C) B_1, B_3 but not B_2

(D) only B_1

25. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and let α_n and β_n denote the two eigen values of A^n such that $|\alpha_n| \geq |\beta_n|$. Then which of the following is not true?

(A) $\alpha_n \to \infty$ as $n \to \infty$

(B) $\beta_n \to \infty$ as $n \to \infty$

(C) det (A^n) = 1 if n is even

(D) det (A^n) = 1 if n is odd

26. Consider the interval (-1,1) and a sequence $\{\alpha_n\}_{n=1}^{\infty}$ of elements in it. Then

(A) Every limit point of $\{\alpha_n\}$ is in (-1,1)

(B) Every limit point of $\{\alpha_n\}$ is in [-1,1]

(C) The limit point of $\{\alpha_n\}$ can only be in {-1,0,1}

(D) The limit point of $\{\alpha_n\}$ cannot be in {-1,0,1}

27. Let S be a surface which is bounded above by the hemisphere $z = 2 + \sqrt{4 - x^2 - y^2}$ and bounded below by the Cone.

$z = \sqrt{x^2 + y^2}$ and $\hat{n}$ is outward unit normal to S.

Let $\vec{F} = x3\hat{i} + y3\hat{j} + xy\hat{k}$ be a vector field, then flux of $\vec{F}$ across the surface S is,

(A) $\frac{176\pi}{13}$ (B) $\frac{152\pi}{17}$ (C) $\frac{176\pi}{15}$ (D) None of these

28. The value of integral $\iint_D \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right)dxdy$, where D is the region in 1st quadrant bounded by hyperbola xy = 1, xy = 9 and the lines y = x, y = 4x, is,

(A) $4+\frac{52}{3}\ell n2$ (B) $8+\frac{52}{3}\ell n2$ (C) $4+\frac{26}{3}\ell n2$ (D) $8+\frac{26}{3}\ell n2$

29. Let G be a group of order 30. The total no. of group isomorphism of G onto itself is,

(A) 7 (B) 8 (C) 17 (D) 24

30. Let A be a closed subset of $\mathbb{R}$. $A \neq \phi$, $A \neq \mathbb{R}$. Then A is

(A) The closure of the interior of A

(B) A countable set

(C) A compact set

(D) Not open

SECTION-B (Q. 31-40): MULTIPLE SELECT QUESTIONs (MSQs)

31. Let V and W be vector space over field F. Let $S:V \to W$ and $S:W \to V$ linear transformations. Then which of the following(s) is (are) not true?

(A) If ST is one to one then S is one-to-one.

(B) If V = W and V is finite-dimensional such that TS = I, then T is invertible.

(C) If dim V = 2 and dim W = 3, then ST is invertible.

(D) If TS is onto, then S is onto.

32. $\{a_n\}$ is a bounded sequence of real numbers, then which of the following is (are) correct ?

(A) $\underset{n}{\text{Inf}}(a_n) \leq \lim_{n\to\infty} \inf(a_n)$ and $\underset{n}{\text{Sup}}(a_n) \leq \lim_{n\to\infty} \text{Sup}(a_n)$

(B) $\lim_{n\to\infty} \inf(a_n) \leq \underset{n}{\text{Inf}}(a_n)$

(C) $\lim_{n\to\infty} \inf(a_n) \le \underset{n}{\text{Inf}}(a_n)$ and $\underset{n}{\text{Sup}}(a_n) \le \lim_{n\to\infty} \text{Sup}(a_n)$

(D) $\underset{n}{\text{Inf}}(a_n) \le \lim_{n\to\infty} \inf(a_n)$ and $\lim_{n\to\infty} \text{Sup}(a_n) \le \underset{n}{\text{Sup}}(a_n)$

33. Let $\sum x_n$ be a series of positive term which is Convergent, then which of the following(s) is/are necessarily true ?

(A) $\sum x_n^2$ is Convergent

(B) $\sum \sqrt{x_n x_{n+1}}$ is Convergent

(C) $\sum x_n^2$ is convergent but $\sum \sqrt{x_n x_{n+1}}$ need not be.

(D) $\sum \frac{\sqrt{x_n}}{n}$ is Convergent

34. There exist a map $f : N \to N$ such that f is

(A) One-one but cannot be on to.

(B) On to but cannot be one-one

(C) One-one and onto.

(D) Cannot be bijective.

35. Let V be the vector space of all real polynomials of degree at most 2. Define a linear operator $T : V \to V$ by

$$T\left(x^i\right) = \sum_{j=0}^{i} x^j,\ i = 0,\ 1,2.$$

Where $\beta = \{1, x, x^2\}$ be the basis for V. Let A = $[T]_\beta$ be the matric of T w.r. to basis β

Which of the followings is/are true ?

(A) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ (B) det T = det(T^{-1} ; Tr(T) = Te(T^{-1})

(C) The dimension of eigen space of T^{-1} Corresponding to eigen value 1 is 2

(D) Characteristic poly and minimal polynomial for T and T^{-1} are same.

36. $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx, B_n \int_0^{\pi/2} \left(\frac{\sin nx}{\sin x}\right) dx$ for $n \in \mathbb{N}$, then

(A) $A_{n+1} = A_n$ (B) $B_{n+1} = B_n$

(C) $A_{n+1}-A_n = B_{n+1}$ (D) $B_{n+1} - B_n = A_{n+1}$

37. Consider the differential equation,

$\left(y + \cos y + \frac{1}{2\sqrt{x}}\right)dx + (x - x \sin y - 1)dy = 0$, then its Solution is given by,

(A) $xy - x\cos y + \sqrt{x} - y = c$

(B) $xy + x\cos y + \sqrt{x} - y = c$

(C) $x - xy \cos y + \sqrt{x} - y = c$

(D) $x + xy \cos y + \sqrt{x} - y = c$

38. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{e^x - e^{-x}}{2}$, $\forall x \in \mathbb{R}$

Then which of the following is/are correct ?

(A) f is one-one but not onto

(B) f is onto but not one-one

(C) f is invertible and $f^{-1}(x) = \ell n\left(y + \sqrt{y^2+1}\right)$

(D) f is invertible and $f^{-1}(x) \ell n\left(x \pm \sqrt{x^2+1}\right)$

39. Let y(x) be the solution of differential equation, $(e^y-x)dx-x^2dy = 0$, $x > 0$ with $y(1) = 0$.

Then which of the following (s) is/are correct ?

(A) $y(x) = \ell n\left(\frac{2x}{1+x^2}\right)$ (B) $y(x) = \ell n\left(\frac{1+x^2}{2x}\right)$

(C) y is bounded (D) y is unbounded

40. If f (x,y) = $\begin{cases} xy\tan\left(\frac{y}{x}\right) & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

Then at (0,0)

(A) x fx + y fy = 2 f (B) x fx - y fy = 2f

(C) y fx + x fy = 2f (D) y fx - x fy = 2f

SECTION-C (Q. 41-60): NUMERICAL ANSWER TYPE's (NATs)

41. If $k = \int_C x^{-1}(y+z)\,ds$, where C is the Arc of the circle $x^2 + y^2 = 4$ in the xy-plane from A(2,0,0) to $B\left(\sqrt{2},\sqrt{2},0\right)$. Then [K] + 1 is equal to________.

42. The greatest value of the directional derivative of the function $\phi = 2x^2 - y - z^4$ at the point (2,-1,1), is__________.

43. Let $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{x^2 - xy}{x+y} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$$

Then $f_x(0,0) + f_y(0,0) =$

44. u = f(x + 2y) + g (x - 2y) , then $\frac{\partial^2 u}{\partial y^2} - 4\frac{\partial^2 u}{\partial x^2} =$

45. The order of subgroup <5> $\oplus$ < 3> in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{12}$ is___________.

46. Let $M_3(\mathbb{R})$ be the vector space of all 3 x 3 real matrices.

LEt V be a subspace of $M_3(\mathbb{R})$ defined by

$$V = \left\{ A \in M_2(\mathbb{R}) \middle/ A\begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} A \right\}.$$

Then dimension of V is________.

47. Let f(x) = $|x^2 - 4|$ for all $x \in \mathbb{R}$. The total number of points on $\mathbb{R}$ at which attains a local extremum (minimum or maximum) is____________.

48. Let $\{(x,y) \in \mathbb{R}^2 : x, y > 0\} \to \mathbb{R}$ be given by

$$f(x,y) = x^{\frac{1}{2}} y^{-\frac{3}{2}} \sin^{-1}\left(\frac{4}{x}\right) + \frac{1}{\sqrt{x^2 + y^2}}$$

Then the value of $\dfrac{x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}}{f(x,y)}$ is equal to__________.

49. Let $\dfrac{d}{dx} = f(x) = \dfrac{e^{\sin x}}{x}$, $x > 0$. If $\displaystyle\int_1^4 \frac{2e^{\sin x^2}}{x} dx = f(k) - f(1)$.

Then one of the possible value of k is_________.

50. Let f(x) be the integrating factor of DE, $\dfrac{dy}{dx} = \dfrac{2xy^2 + y}{x - 2y^3}$, then f(1)=_____.

51. Solution of the equation $x^2 - x + 1 = \dfrac{1}{2}\sqrt{x - \dfrac{3}{4}}$, where $x \geq \dfrac{3}{4}$ is equal to____.

52. Let $K\sqrt{2}\pi$ be the surface area of the portion of the Cone $z^2 = x^2 + y^2$ that is inside the cylinder $z^2 = 4y$.

then the value of K is__________.

53. The Sum of $1+\frac{2}{1.2.3}+\frac{2}{3.4.5}+\frac{2}{5.6.7}+\frac{2}{7.8.9}+..........$ is (correct upto three decimal places)

54. Let $K=\frac{1}{1!}+\frac{1+2}{2!}+\frac{1+2+2^2+2^3}{4!}+..........$

Then [K] is equal to.......([.] denotes greatest integer function)

55. Let S and T be two subspace of $\mathbb{R}^{24}$ Such that dim S = 19 and dim T = 17. Then the smallest possible value of dim (S + T) is_________.

56. Let ξ be a primitive fifth root of unity. Define

$$A=\begin{bmatrix} \xi^{-2} & 0 & 0 & 0 & 0 \\ 0 & \xi^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{bmatrix}$$

for a vector V = $(V_1, V_2, V_3, V_4, V_5) \in \mathbb{R}^5$ define

$|V|_A=\sqrt{|VAV^T|}$, where V^T is Transpose of V. If W = (1, -1, 1, 1, -1) then $|W|^A$ is equal to__________.

57. The area enclosed between the curves y = log (x + e), $x=\log\frac{1}{y}$ and the x-axis is____________.

58. The area of surface of the paraboloid $\frac{x^2}{a}+\frac{y^2}{b}=2z$ inside the cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2}=K$ is given by mab $[(1 + K)^{3/2} - 1]$, then m = ...(correct upto two decimal places).

59. Let $\bar{F} = (2x^2 - 3z)i - 2xy\,j - 4xk$ be a vector field and V is the closed region bounded by the planes x = 0, y = 0, z = 0 and 2x + 2y + z = 4. If $\bar{K} = \int_V \iint (\nabla x \vec{F})\,dv$. Then $|\bar{K}| = ...$ (correct up to three decimal places).

60. The no. of elements in the conjugacy class of the 3-cycle (2,3,4) in the symmetric S_6 is__________.

ANSWER KEY FMTP

Ques	1	2	3	4	5	6	7	8	9	10
Ans	C	B	A	C	A	D	C	D	D	B
Ques	11	12	13	14	15	16	17	18	19	20
Ans	C	D	A	C	B	C	D	D	D	B
Ques	21	22	23	24	25	26	27	28	29	30
Ans	C	A	A	A	D	B	C	A	B	A
Ques	31	32	33	34	35	36	37	38	39	40
Ans	A,C,D	D	A,B,D	C	A,B,D	A,D	B	C	D	A,B,C,D
Ques	41	42	43	44	45	46	47	48	49	50
Ans	1	9	1	0	24	3	3	-1	16	1
Ques	51	52	53	54	55	56	57	58	59	60
Ans	1	4	1.386	4	19	0	2	0.66	3.771	40

HINTS & SOLUTION

1.(C)

Sol. The last term $\frac{1}{n}=\frac{n+1}{n^2+1^2}$

Thus $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{n+r}{n^2+r^2} = \lim\limits_{n\to\infty}\sum_{r=1}^{\infty}\frac{1}{n}\frac{1+\frac{r}{n}}{1+\left(\frac{r}{n}\right)^2}$

$$=\int_0^1 \frac{1+x}{1+x^2}\,dx$$

$$=\int_0^1 \frac{1}{1+x^2}\,dx+\int_0^1\frac{x}{1+x^2}\,dx$$

$$=\left(\tan^{-1}\right)_0^1+\frac{1}{2}\left[\log\left(1+x^2\right)\right]_0^1$$

$$=\frac{\pi}{4}+\frac{1}{2}\log 2\in Q^C$$

2.(B)

Sol. Let $\phi = x^2y + 2xz - 4$

$$\nabla\phi = (2xy + 2z)i + x^2j + 2xk$$

$$\nabla\phi\Big|_{(2,-2,3)} = -2i + 4j + 4k$$

$$\Rightarrow \hat{n} = -\frac{1}{3}i + \frac{2}{3}j + \frac{2}{3}k \text{ or } \frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}k$$

3.(A)

Sol. Let $I = \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$ ____________(1)

$$I = \int_0^{\pi} \frac{e^{\cos(\pi - x)}}{e^{\cos(\pi-x)} + e^{-\cos(\pi-x)}} dx$$

$$= \int_0^{\pi} \frac{e^{-\cos x}}{e^{-\cos x} + e^{\cos x}} dx \text{ ____________(2)}$$

adding (!) and (2) , we get

$$2I = \int_0^{\pi} dx = \pi$$

$$\Rightarrow I = \frac{\pi}{2}$$

4.(C)

Sol. Given $\frac{dy}{dx} = \frac{y^3}{e^{2x} + y^2}$

$$\Rightarrow \frac{dx}{dy} = \frac{e^{2x} + y^2}{y^3}$$

$$\Rightarrow \frac{dx}{dy} = \frac{e^{2x}}{y^3} + \frac{1}{y} \Rightarrow \frac{dx}{dy} - \frac{1}{y} = \frac{e^{2x}}{y^3}$$

$$\Rightarrow \qquad e^{-2x}\frac{dx}{dy}-\frac{e^{-2x}}{y}=\frac{1}{y^3} \quad \_\_\_\_\_\_\_\_\_\_(1)$$

So by (1),

$$-\frac{1}{2}\frac{dv}{dy}+\frac{2v}{y}=-\frac{2}{y^3} \qquad \text{(LDE)}$$

I.F. = $e^{2\ell ny}=y^2$

Sol^n : $v\,y^2=-2\int\frac{1}{y}\,dy+c$

$$v\,y^2=-2\log y+c$$

$$\Rightarrow y^2e^{-2x}=-2\log y+c$$

Given curve passing through (0,1)

$\Rightarrow$ C = 1

$$\Rightarrow y^2e^{-2x}=-2\log y+1$$

$$\Rightarrow e^{-2x}=\frac{1}{y^2}\log\frac{1}{y^2}+\frac{1}{y^2}=\frac{1}{y^2}\left(\log\frac{1}{y^2}+\log e\right)$$

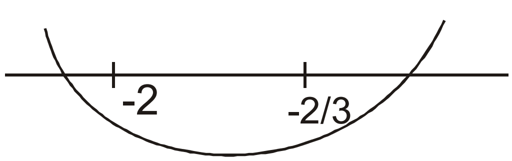
$$=\frac{1}{y^2}\log\frac{e}{y^2}$$

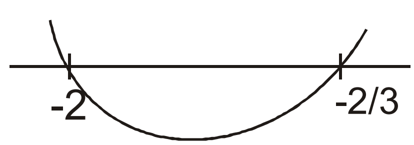
5.(A)

Sol. For f to be decreasing we want f'(x) ≤ 0

$$\Rightarrow 3x^2+8x+\lambda\le 0,\ \forall x\in\left(-2,-\frac{2}{3}\right)$$

The situation for f'(x) are as follows





Given that f is dec. in largest possible int. $\left(-2,-\frac{2}{3}\right)$.

Then f'(x) = 0 must have roots -2 and $-\frac{2}{3}$

Thus product of roots = $=(-2)\left(-\frac{2}{3}\right)=\frac{\lambda}{3}\Rightarrow \boxed{\lambda=4}$

6.(D)

Sol. Basis for V : {1, x, x^2, x^3}

T(1) = 1

T(x) = x + 1

T(x2) = x^2 + 2x - 2

T(x3) = x^3 + $3x^2$ - 6x

Thus basis for R(T) : (1, x+1, x^2 + 2x - 2, x^3 + $3x^2$ - 6x}

$\Rightarrow$ Rank T = 4

7.(C)

Sol. Let $A'=\begin{bmatrix} a & a \\ a & a \end{bmatrix}$ be the inverse of A. Then

A'A = AA' = I

Where $I=\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ is multiplicative let of S

Then $\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} a & a \\ a & a \end{bmatrix}=\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

$\Rightarrow\begin{bmatrix} 6a & 6a \\ 6a & 6a \end{bmatrix}=\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$

$$\Rightarrow a = \frac{1}{12}$$

$$\Rightarrow A' = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$ **Ans.**

8.(D)

Sol. (A) Here $u_n = \frac{(2n-1)(2n)}{(2n+1)^2(2n+1)^2}$: Cgt by Comparison test.

(B) $u_n = \frac{n+1}{n^p} = \frac{1+\frac{1}{n}}{n^p - 1}$: Cgt by C.T.

(C) $u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$

$$= \frac{2}{\sqrt{n^4+1}+\sqrt{n^4-1}} = \frac{2}{n^2\left[\sqrt{1+\frac{1}{n^4}}+\sqrt{1-\frac{1}{n^4}}\right]}$$: Cgt by C.T.

9.(D)

Sol. For non-trivial sol^n det of Coefficient, matrix should be 0.

$$\Rightarrow \begin{vmatrix} 1 & K & 3 \\ 3 & K & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ -4K - 4K + 27 - 6K + 6 + 12K = 0

$\Rightarrow$ -2K + 33 = 0

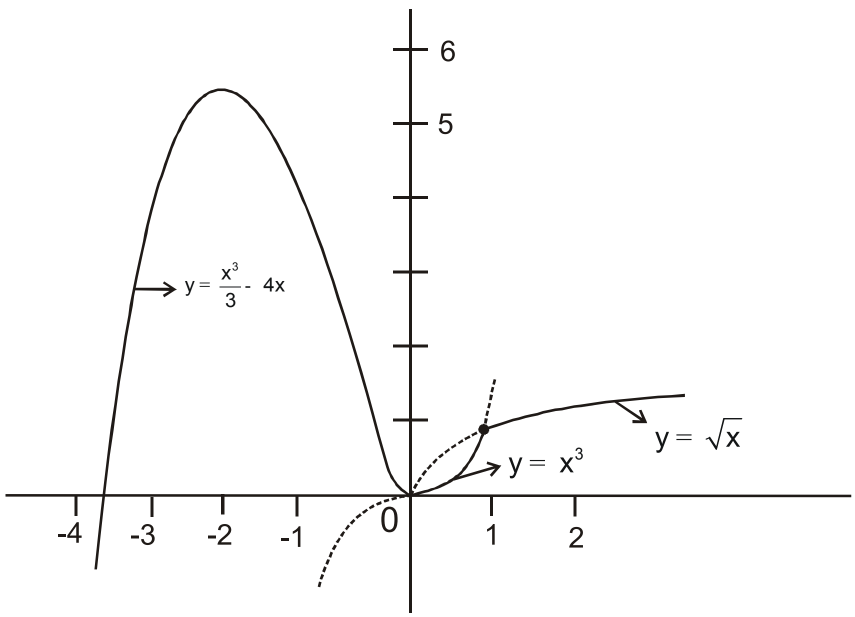
$$\Rightarrow K = \frac{33}{2}$$ **Ans.**

10.(B)

Sol. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}e^{\frac{r}{n}}=\int\limits_0^1 e^x\,dx=\left(e^1-1\right)$

11.(C)

Sol. The graph of f is given as,



By graph it is clear that f is increasing in $\left(-\infty,-2\right)\cup\left(0,\infty\right)$ and decreasing in (-2,0).

x = -2 is local maxima and x = 0 is local minima.

It is derivable $\forall x\in\mathbb{R}-\{0,1\}$ and continuous $\forall x\in\mathbb{R}$.

12.(D)

Sol. Let $I=\int\limits_0^{\pi}\frac{\sin\left(2x\right).\sin\left(\frac{\pi}{2}\cos x\right)dx}{2x-\pi}$ ________________(1)

$\Rightarrow\quad I=\int\limits_0^{\pi}\frac{\left(\pi-x\right)\sin\left(2\pi-2x\right)\,\sin\left(\frac{\pi}{2}\cos\left(\pi-x\right)\right)dx}{2\left(\pi-x\right)-\pi}$

$$= \int_0^{\pi} -\frac{(\pi - x)\sin(2x).\sin\left(\frac{\pi}{2}\cos x\right)dx}{(2x-\pi)} \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

adding (1) (2) we get, we get,

$$2I = \int_0^{\pi} \frac{(2x-\pi)\sin(2x).\sin\left(\frac{\pi}{2}\cos x\right)dx}{(2x-\pi)}$$

$$= \int_0^{\pi} \sin(2x)\sin\left(\frac{\pi}{2}\cos x\right)dx = 2\int_0^{\pi} \sin x\cos x.\sin\left(\frac{\pi}{2}\cos x\right)dx$$

Let $\frac{\pi}{2}\cos x = t$

$$\Rightarrow -\frac{\pi}{2}\sin x\, dx = dt \Rightarrow \sin dx = -\frac{2}{\pi}$$

$$\Rightarrow 2I = -\frac{2.2}{\pi}\int_{+\pi/2}^{-\pi/2} \frac{2}{\pi}t\sin t\, dt$$

$$\Rightarrow I = \frac{4}{\pi^2}\int_{-\pi/2}^{\pi/2} t\sin t\, dt$$

$$= \frac{8}{\pi^2}\int_{-\pi/2}^{\pi/2} t\sin t\, dt = \frac{8}{\pi^2}\left[-t\cos t + \sin t\right]_0^{\pi/2}$$

$$= \frac{8}{\pi^2} \quad \Rightarrow \boxed{I = \frac{8}{\pi^2}}$$

13.(A)

Sol. We know Rank A = Rank A^T= 2

and also we know Rank (AB) $\leq$ min {Rank A, Rank B} as here we are taking product of matrix with its transpose.

Thus Rank (A^TA) = 2

14.(C)

Sol. $f(x) = x^2(x - 1)$

Then f is twice continuous diff. function on $\mathbb{R}$

with $f(0) = f(1) = f'(0) = 0$

and here $f''(x) = 2x - 2 \neq 0$

and $f''(0) = = -2 \neq 0$

and $f''(1) = 0$ and $1 \in [0, 1]$

15.(B)

Sol. Last term in $\dfrac{1}{\sqrt{2n}+\sqrt{2n+2}}$

$$= \frac{\sqrt{2n}-\sqrt{2n+2}}{(2n)-(2n+2)} = \frac{1}{2}\left(\sqrt{2n+2}+\sqrt{2n}\right)$$

Thus $= \dfrac{1}{\sqrt{n}}\left(\dfrac{1}{\sqrt{2}+\sqrt{4}}+\dfrac{1}{\sqrt{4}+\sqrt{6}}+\text{..........}+\dfrac{1}{\sqrt{2n}+\sqrt{2n+2}}\right)$

$$=\frac{1}{\sqrt{n}}\left(\frac{1}{2}\left(\sqrt{4}-\sqrt{2}\right)+\frac{1}{2}\left(\sqrt{6}-\sqrt{4}\right)+.....+\frac{1}{2}\left(\sqrt{2n}-\sqrt{2n-2}\right)+\frac{1}{2}\left(\sqrt{2n+2}-\sqrt{2n}\right)\right)$$

$$= \frac{1}{2\sqrt{n}}\left(-\sqrt{2}+\sqrt{2n+2}\right)$$

$$= -\frac{1}{\sqrt{2n}}+\frac{\sqrt{2}}{2}\sqrt{1+\frac{1}{n}} \to \frac{1}{\sqrt{2}} \text{ as } n \to \infty$$

16.(C)

Sol. Write down the given Non-Homo. eq^n in standard form as

$$y''+\frac{(x-2)}{x}y'-\frac{2}{x}y = x^2, \text{ \_\_\_\_\_\_\_\_\_(1)}$$

Then by variation of parameter method general Sol^n of (1) is given by,

$y(x) = C_1y_1 + C_2y_2 + e^{-x} f(x) + (x^2 - 2x + 2) g(x)$ ___________(2)

Where $f(x) = -\int \frac{x^2.(x^2 - 2x + 2)}{x^2 e^{-x}} dx$ (where $x^2e^{-x} = w(y_1, y_2)$)

$= -\int e^x (x^2 - 2x + 2) dx$

$= - e^x (x^2 - 4x + 6)$

and $g(x) = \int \frac{e^{-x}x^2}{x^2 e^{-x}} dx = x$

So by (2), we have

$y(x) = C_1e^{-x} + C_2 (x^2 - 2x + 2) - (x^2 - 4x + 6) + x (x^2 - 2x + 2)$

$\Rightarrow$ $(V(x) = (x^3 - 3x^2 + 6x - 6)$

$\Rightarrow$ $V(0) = -6$

17.(D)

Sol. Since $f_1(x) = x$ is the identity element of group

Hence $(f_3 \; 0 \; f_3)(x) = f_3(f_3(x))$

$f_3 = \left(\frac{1}{x}\right) = x$

$\Rightarrow$ f_3 is self invertible

18.(D)

Sol. We have

$G = \{...., -3, -2, -1, 0, 1, 2, 3, ...\}$

and $H = 4\mathbb{Z} = \{,......., 12, -8, -4, 0, 4, 8, 12, ...\}$

Then the cosets of H by 0 is

$H_0 = \{h + 0 / h \in H\}$

$= \{......, -12, -8, -4, 0, 4, 8, 12, ...\}$

The coset of H by 1 is

$H_1 = \{h + 1 ; h \in H\}$

$\{......., -7,, -3, 2,, 5, 9, ...\}$

The coset of h by 2 is

$H_2 = \{h + 2 / h \in H\} = \{......, -6, -2, 2, 6, 10, ...\}$

Similarly $H_3 = \{h + 3 \ / \ h \in H\} = \{......, -5, -1, 3, 7, 11, ...\}$

$H_4 = \{h + 4 \ / \ h \in H\} = \{......, -4, 0, 4, 8, 12, ...\} = H_0$

Hence H has four distinct coset in G.

19.(D)

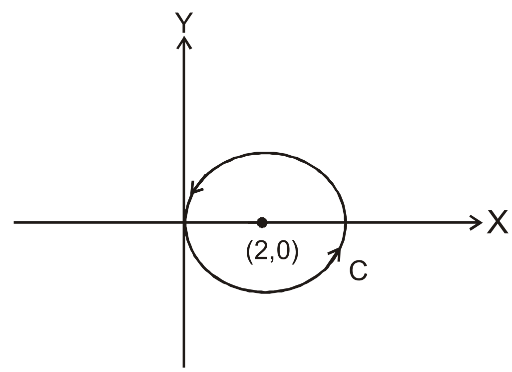
Sol. S : $x^2 + y^2 + z^2 - 4x + 2z = 0 \quad z \geq 0$

Then Bounding Curve of S is given by

C : $x^2 + y^2 - 4x = 0 \Rightarrow (x - 2)^2 + y^2 = 4$

as here S be an open surface, So we can use S take's thm

To compute $\iint_S \left(\nabla x\vec{F}\right).\hat{n}ds$.



So By S. T. $\iint_S \left(\nabla x\vec{F}\right).\hat{n}ds = \oint_C \bar{F}.d\bar{r}$

$= \oint_C \left(y^2 + x^2\right)dx + \left(yx^2 + y^2\right)dy$

$= \iint \left(2xy - 2y\right)dxdy$ (By G.T.

$= 2\iint y\left(x - 1\right)dxdy$

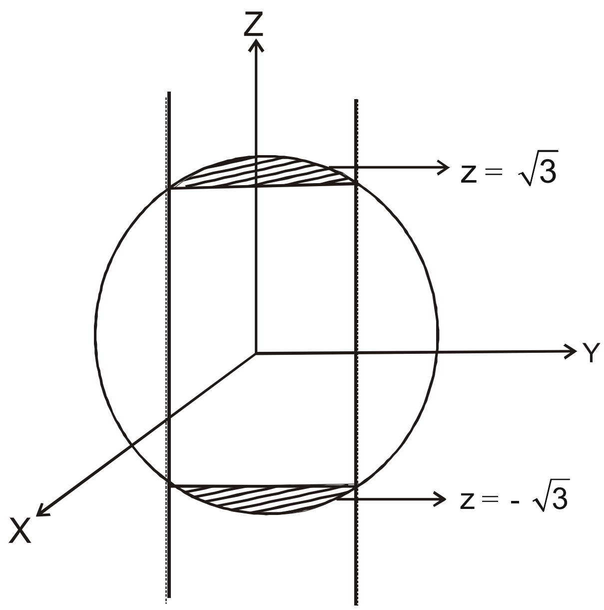
= 0 **Ans.**

20.(B)

Sol. $\hat{n} xi + yj$

$\Rightarrow \bar{F}.\hat{n} = x^2y + y^2z^2$

$\Rightarrow \iint \bar{F}.\hat{n}ds = \iint_S (x^2y + y^2z^2)ds$ __________**(1)**



$x^2 + y^2 + z^2 = 4$; $x^2 + y^2 = 1$

$\Rightarrow \Rightarrow z = \pm\sqrt{3}$

Use Cylinderical coordinate to evaluate (1),

i.e. $x = \cos\theta$, $y = \sin\theta$

$ds = dzd\theta$

Where $\theta: 0$ to 2π and $z: -\sqrt{3}$ to $\sqrt{3}$

$$\Rightarrow \iint \bar{F}.\hat{n}ds = \int_{\theta=0}^{2\pi}\int_{z=-\sqrt{3}}^{\sqrt{3}} (\cos^2\theta\sin\theta + \sin^2\theta z^2)d\theta dz$$

$$= \int_{\theta=0}^{2\pi}\int_{z=-\sqrt{3}}^{\sqrt{3}} \sin^2 z^2\, dzd\theta\, d\theta$$

$$= 2.4 \int_0^{\pi/2} \sin^2\theta \left(\frac{z^3}{3}\right)_0^{\sqrt{3}} d\theta$$

$$= \frac{8}{3} 3\sqrt{3} \frac{\sqrt{\frac{3}{2}}\sqrt{\frac{1}{2}}}{2\sqrt{2}} = 2\sqrt{3}\,\pi$$ **Ans.**

21.(C)

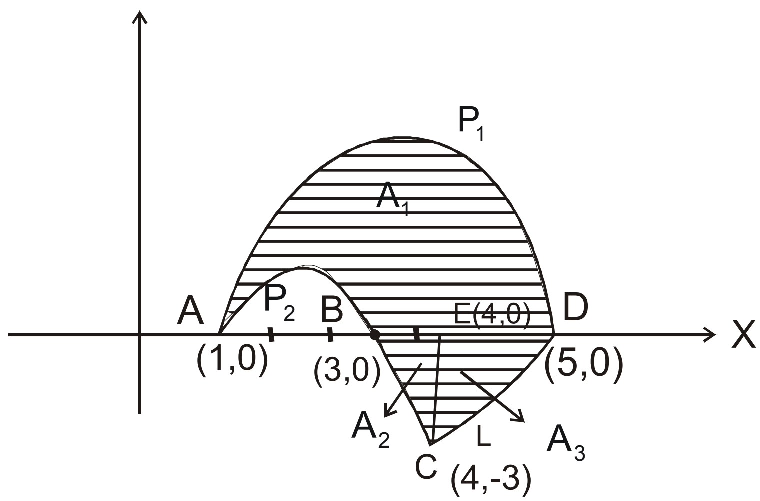
Sol. The two parabola's can be re-written as and they cut x-axis at points noted.

P_1 : $(x - 3)^2 = -(y - 4)$, (1,0) and (5,0)

P_2 : $(x - 3)^2 = -(y - 4)$, (1,0) and (5,0)

L : $y = 3x - 15$, (5, 0)

(1,0) is the common point of



P_1 and P_2 ; (5,0) is common of

P_1 and L ; (4,-3) is common of

P_2 and L

The required Area = $A_1 + A_2 + A_3$

Where $A_1 = \int_1^5 y_1\,dx - \int_1^3 y_2\,dx = \int_1^5 \left(-x^2 + 6x - 5\right)dx - \int_1^3 \left(-x^2 + 4x - 3\right)dx$

$$A_2 = \left|\int_3^4 y_2\,dx\right| = \left|\int_3^4 \left(-x^2 + 4x - 3\right)dx\right| = \left|-\frac{4}{3}\right| = \frac{4}{3}$$

$$A_3 = \left|\int_4^5 (3x - 15)\right| = \left|-\frac{3}{2}\right| = \frac{3}{2}$$

So required Area = $= \frac{28}{3} + \frac{4}{3} + \frac{3}{2} = \frac{56+8+9}{6} = \frac{73}{6}$ **Ans.**

22.(A)

Sol. Given $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... + \frac{(-1)}{n} + ...\log 2$ _________(1)

Let $u_n = \frac{(-1)^n}{n(n+1)} = (-1)^n\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$\Rightarrow \sum_{n=1}^{\infty}\frac{(-1)^n}{n(n+1)} = (-1)^n\left[\frac{1}{n} - \frac{1}{n+1}\right]$

$= \sum_{n=1}^{\infty}\frac{(-1)^n}{n} - \sum_{n=1}^{\infty}\frac{(-1)^n}{(n+1)}$

$= \left[-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + ...\right] - \left[-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}........\right]$

$= -\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right] - \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... - 1\right]$

= - log 2 - [log 2 - 1]

= 1 - 2 log 2 **.**

23.(A)

Sol. Given T(p(t)) = Q(t) = P(at + b)

$\Rightarrow$ T(1) = 1

T(t) = at + b

$T(t^2) = (at + b)^2 = a^2t^2 + 2abt + b$

$\Rightarrow [T]\begin{bmatrix} 1 & b & b^2 \\ 0 & a & 2ab \\ 0 & 0 & a^2 \end{bmatrix}$

24.(A)

Sol. Clearly B_1 and B_2 and L.I. subsets of $\mathbb{R}^4$. Hence basis now check for B_3.

$$\det\left(\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ -5 & 5 & 0 & 0 \end{bmatrix}\right) = 1\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 5 & 0 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 0 \\ -5 & 0 & 0 \end{vmatrix}$$

= 0 - 2(0) = 0

$\Rightarrow$ B_3 not L.I. Hence not basis for $\mathbb{R}^4$.

25.(D)

Sol. Char. eqn A : $x^2 - x - 1 = 0$

$$\Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$$

Let $\lambda_1 = \frac{1+\sqrt{5}}{2}$ and $\lambda_2 = \frac{1-\sqrt{5}}{2}$

Then eigenvalues of A^n are $(\lambda_1)^n$ and $(\lambda_2)^n$

Clearly $|\lambda_1| \geq |\lambda_2|$

$\Rightarrow \alpha_n = \lambda_1^n$ and $\beta_n = \lambda_1^0$

$$\Rightarrow (\alpha_n) = \left(\frac{1+\sqrt{5}}{2}\right)^n \text{ and } (\beta_n) = \left(\frac{1-\sqrt{5}}{2}\right)^n$$

Clearly $\lim\limits_{nx \to \infty} \alpha_n = \infty$ and $\lim\limits_{nx \to \infty} \beta_n = 0$

also β_n will be +ive when n is even.

Now det (A^n) = product of ev's

$$= \left(\frac{1+\sqrt{5}}{2} \cdot \frac{1+\sqrt{5}}{2}\right)^n = \left(\frac{(1-4)}{4}\right)^n = (-1)^n$$

26.(B)

Sol. Let $\{\alpha_n\} = 1 - \frac{1}{n}$, $n \in \mathbb{N}$

Then $\{\alpha_n\} \in (-1,1)$ and $\lim_{n\to\infty} \alpha_n = 1 \Rightarrow$ 1 is the only L.P. of $\{\alpha_n\}$

But $1 \notin (-1,1)$

again Let $\alpha_n = \frac{1}{2} - \frac{1}{n} . \forall n \in \mathbb{N}$

Then also $\alpha_n \in (-1,1)\ \forall n$ and $\lim_{n\to\infty} \alpha_n = \frac{1}{2}$, So $\frac{1}{2}$ be the only L.P. of $\{\alpha_n\}$ but

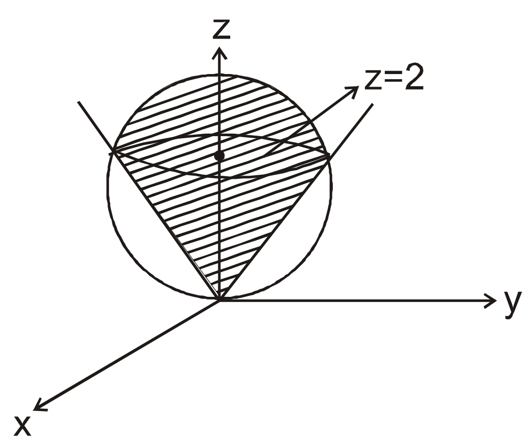
$\frac{1}{2} \notin \{-1,0,1\}$

$\Rightarrow$ (a), (c) and (d) cannot be true

Thus only (b) is true.

27.(C)

Sol. Point of intersection :



$z = 2 + \sqrt{4 - z^2}$

$\Rightarrow z^2 - 4z + 4 = 4 - z^2$

$\Rightarrow 2z^2 - 4z = 0 \Rightarrow z = 0$ or 2

i.e. at z = 2 we have a curve of int. as x2 + y2 = 4

Now flux $= \oiint_S \bar{F}.\hat{n}ds$

$$= \iiint_V (\nabla.\bar{F})dv \quad \text{(By GDT.)}$$

Now $\nabla.\bar{F} = 3x^2 + 3y^2$

$$\Rightarrow \iiint (\nabla.\bar{F})dv = 3\iiint (x^2 + y^2)dv$$

$$= 3\iint (x^2 + y^2)\left[2 + \sqrt{x^2 - y^2} - \sqrt{x^2 + y^2}\right] dxdy$$

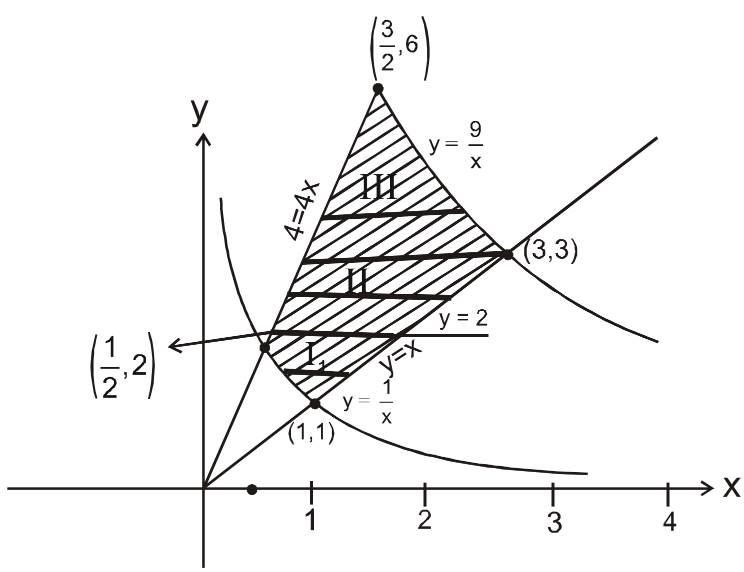
$$= 2\pi\int_0^2 \left(2r^3 + r^3\sqrt{4 - r^2} - r^4\right)dr$$

$$= 2\pi\left[8 - \frac{32}{5} + \int_0^2 (4t^2 - t^4)dt\right]$$

$$= 2\pi\left[\frac{8}{5} + \frac{32}{3} - \frac{32}{5}\right] \quad = 2\pi \, x\frac{88}{15} = \frac{176\pi}{15}$$ **Ans.**

28.(A)

Sol.



$$y=\frac{1}{x}, y=\frac{9}{x}$$

$$\Rightarrow \iint_D \left(\sqrt{\frac{y}{x}}+\sqrt{xy}\right)dxdy$$

$$=\iint_{\text{I}} + \iint_{\text{II}} + \iint_{\text{III}}$$

On I $\int_{y=1}^{2}\int_{x=\frac{1}{y}}^{y}\left(\sqrt{\frac{y}{x}}+\sqrt{xy}\right)dxdy$

$$=\int_1^2\left[y^{1/2}\frac{x^{1/2}}{1/2}+y^{1/2}\frac{x^{3/2}}{3/2}\right]dy=\int_1^2\left[2y^{1/2}y^{1/2}+\frac{2}{3}y^{1/2}y^{3/2}-2-\frac{2}{3}\ \frac{1}{y}\right]dy$$

$$=\int_1^2\left(2y+\frac{2}{3}y^2-2-\frac{2}{3}\ \frac{1}{y}\right)dy$$

$$=\left[y^2+\frac{2}{9}y^3-2y-\frac{2}{3}\ell ny\right]_1^2$$

$$=4+\frac{16}{9}-8-\frac{2}{3}\ell n2-\left(1-\frac{2}{9}-2\right)$$

$$=-\frac{13}{9}-\frac{2}{3}\ell n2$$

On II : $=\int_1^2\left[2y^{1/2}x^{1/2}+\frac{2}{3}y^{1/2}x^{3/2}\right]_{y/4}^{y} dy$

$$=\frac{223}{36}$$

On III $:=\int_3^6\left[2y^{1/2}x^{1/2}+\frac{2}{3}y^{1/2}x^{3/2}\right]_{y/4}^{y} dy$

$$= \int_3^6 \left(6 + \frac{18}{y} - y - \frac{1}{12}y^2\right) dy$$

$$= 18\ell n2 + \frac{21}{4} - 6$$

$$\Rightarrow \iint_D = 4 + \frac{52}{3}\ell n2$$ **Ans.**

29.(B)

Sol. G be a cyclic group of order 30.

Total no of generator of $G = \phi(30)$

$$= \phi(2x3x5)$$

$$= 30\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{5}\right)$$

$$= 30x\frac{1}{2}x\frac{2}{3}x\frac{4}{5}$$

= 8

Hence thn no. of group iso of G onto itself is 8

30.(A)

Sol. Let A = {1,2,3,.....} = **N**

Then $A \subset \mathbb{R}$ with $A \neq \phi, A \neq \mathbb{R}$ and A is closed.

But A is not compact as A is not bdd. also A is not open as no point of A be its interior point.

$\Rightarrow$ (c) and (d) not true

or if we take A = [a,b] , $a,b \in \mathbb{R}, a < b$

Then A is not true.

$\Rightarrow$ (b) also not true

$\Rightarrow$ only (a) is true.

31. (A,C,D)

Sol. (A)Let $S : \mathbb{R}^3 \to \mathbb{R}^2$ by

$S(x_1, x_2, x_3) = (x_1, x_2)$

and $T : \mathbb{R}^2 \to \mathbb{R}^3$ by

$T(x_1, x_2) = (x_1, x_2, x_1 + x_2)$

Then $ST : \mathbb{R}^2 \to \mathbb{R}^2$ by

$ST(x_1, x_2) = S(x_1, x_2, x_1 + x_2)$
$= (x_1, x_2)$

$\Rightarrow ST = I \Rightarrow ST$ is invertible, i.e. one-one and onto byt S not one-one.

$\Rightarrow$ (A) not true.

(B) Given V = W, V is FDVS

Then S, T : V $\to$ V are L.T. S.T. $TS = I$

$\Rightarrow$ TS is invertible i.e. one-one and onto.

as T : V $\to$ V , V is FDVS and T is onto

$\Rightarrow$ T is one-one

$\Rightarrow$ T is invertible

$\Rightarrow$ (b) is true.

(C) Let $V = \mathbb{R}^2$ and $W = \mathbb{R}^3$

Then $S : \mathbb{R}^2 \to \mathbb{R}^3$, $T : \mathbb{R}^3 \to \mathbb{R}^2$ by

S(x,y) = (x,x,y)

T(x,y,z) = (x + y, y + z)

Then $ST : \mathbb{R}^3 \to \mathbb{R}^3$ ST(x,y,z) = (x + y, x+y, y+z) clearly ST is not one-one, hence not invertible.

(D) we can easily check (d) also not true.

32.(D)

Sol. Take $a_n = \frac{1}{n}$ then $\underset{n}{\text{Sup}}(a_n) = 1$

and $\lim_{n\to\infty} \text{Sup}(a_n) = 0 = \lim_{n\to\infty} \inf(a_n)$

$\Rightarrow$ (a) and (c) are Contradiction.

Again take $a_n = -\frac{1}{n}$ then $\lim_{n\to\infty} \inf(a_n) = 0$ and $\underset{n}{\text{Sup}}(a_n) = -1$

$\Rightarrow$ (b) are Contradiction.

$\Rightarrow$ only (d) is correct

33. (A,B,D)

Sol. Given that $x_n > 0\ \forall_n$ and $\sum x_n$ is Convergent $\Rightarrow \lim_{n\to\infty} x_n = 0$

So choose $\in = 1$. Then $\exists n_o \in \mathbb{N}$ S.T. $|x_n - 0| < 1\ \forall_n \geq n_0$

$\Rightarrow |x_n| < 1,\ \forall_n \geq n_0$

$\Rightarrow 0 < x_n < 1,\ \forall_n \geq n_0$

$\Rightarrow o < x_n^2 < x_n < 1,\ \forall_n \geq n_0$

So then by Comparison test , as $\sum x_n$ Cgt

$\Rightarrow \sum x_n^2$ Cgt

Again for positive α, β, We have $\sqrt{\alpha\beta} \leq \frac{\alpha+\beta}{2}$ ($\because$ GMS A.M

$\because x_n > 0,\ x_{n+1} > 0\ \ \forall n$

$\Rightarrow \sqrt{x_n\ x_{n+1}} \leq \frac{x_n + x_{n+1}}{2}$

Now since $\sum x_n$ and $\sum x_{n+1}$ both are Cgt

$\Rightarrow \sum x_n x_{n+1}$ also Cgt.

as G.M $\leq$ A.M.

$$\Rightarrow \sqrt{\frac{x_n}{n}} = \sqrt{x_n.\frac{1}{n^2}} \le \frac{x_n + \frac{1}{n^2}}{2} = \frac{x_n}{2} + \frac{1}{2n^2}$$

again $\sum x_n$ and $\sum \frac{\sqrt{x_n}}{n}$ is Cgt.

34.(C)

Let $f : N \to N$, Defined by

f (x) = x - $(-1)^x$

$\Rightarrow$ f (1) = 2

f (2) = 1

f (3) = 4

f (4) = 4 - (1) = 3

f (5) = 5 - (1) = 5 .....so on

$$f(x) = \begin{cases} x-1, & x \text{ is even} \\ x+1, & x \text{ is odd.} \end{cases}$$

$\Rightarrow$ Clearly f is one-one and onto.

$\Rightarrow$ Option (a) , (b) , (d) not true. and option (c) is true.

35. (A,B,D)

Sol. T(1) = 1

T(x) = 1 + x

T(x^2) = 1 + x + x^2

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

and det T = 1 , $\begin{matrix} Tr\left(T^{-1}\right) \\ \text{"} \\ Tr\left(T\right) = 3 \end{matrix}$, det T^{-1} = 1

Now Char eq^n for T^{-1} : $\left(A^{-1}-\lambda I\right)=0$

$$\Rightarrow \left(1-\lambda\right)^3=0 \Rightarrow \lambda=1,1,)$$

Now no. of L.I. $\overline{ev}$ Corr. to repeated ev $\lambda=1=$ nullity $\left(A^{-1}-\lambda I\right)=1$

$\Rightarrow$ dim. of eign space of T^{-1} Corr. to $\lambda=1$ is 1

we can easily verify that minimal poly and Char. Poly for T and T^{-1} is $(1-x)^3$.

36. (A,D)

Sol. $$A_{n+1}-A_n=\int_0^{\pi/2}\frac{\sin(2n+1)x-\sin(2n-1)x}{\sin x}dx$$

$$=\int_0^{\pi/2}2\cos nx \; dx=0$$

$\Rightarrow A_{n+1}=A_n$

$$B_{n+1}-B_n=\int_0^{\pi/2}\frac{\left(\sin(n+1)x\right)^2-\left(\sin nx\right)^2}{\sin^2 x}dx$$

$$=\int_0^{\pi/2}\frac{\sin(2n+1)x}{\sin x}dx=A_{n+1}$$

37.(B)

Sol. Given DE can be written as

$$\left(ydx+xdy\right)+\left(\cos y\,dx-x\sin y\,dy\right)+\frac{1}{2\sqrt{x}}dx-dy=0$$

$$\Rightarrow d\left(xy\right)+d\left(x\cos y\right)+\frac{1}{2\sqrt{x}}dx-dy=0$$

$\Rightarrow xy+x\cos y+\sqrt{x}-y=c$ **Ans.**

38.(C)

Sol. Let $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$

$$\Rightarrow e^{x_1} < e^{x_2} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_} (1)$$

and $e^{-x_2} < e^{-x_1}$ ____________(2)

adding (1) and (2), we get

$\Rightarrow e^{x_1} + e^{-x_2} < e^{x_2} + e^{-x_1}$

$$\Rightarrow \frac{e^{x_1} - e^{-x_1}}{2} < \frac{e^{x_2} - e^{-x_2}}{2} \Rightarrow f(x_1) < f(x_2)$$

$\Rightarrow$ f is strictly Inc. in $\mathbb{R}$

$\Rightarrow$ f is one-one.

again when $x \to -\infty$ then $f(x) \to -\infty$

when $x \to \infty$ then $f(x) \to \infty$

$\Rightarrow$ Range $(f) = \mathbb{R} \Rightarrow$ f is onto

$\Rightarrow$ f is Invertible

for f^{-1}: Let $y = \dfrac{e^x - e^{-x_1}}{2}$

$$\Rightarrow 2y = e^x - e^{-x} \Rightarrow e^{2x} - 2ye^{-x} - 1 = 0$$

$$\Rightarrow e^x = \frac{2y \pm \sqrt{4y^2 + 40}}{2}$$

$$= y \pm \sqrt{1 + y^2}$$

$$\Rightarrow e^x = y\sqrt{1 + y^2} \qquad (\text{as } y - \sqrt{1 + y^2} < 0$$

$$\Rightarrow x = \ell n\left(y + \sqrt{1 + y^2}\right)$$

39.(D)

Sol. Re write DE as, $\left(e^{y}-x\right)-x^{2}\dfrac{dy}{dx}=0$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{y}-x}{x^{2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{e^{y}}{x^{2}}-\frac{1}{x}$$

$$\Rightarrow e^{-y}\frac{dy}{dx}+\frac{e^{-y}}{x}=\frac{1}{x^{2}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Let $e^{-y}=V \Rightarrow -e^{-y}\dfrac{dy}{dx}=\dfrac{dv}{dx}$

So by (1) we have,

$$-\frac{dv}{dx}+\frac{v}{x}=\frac{1}{x^{2}} \Rightarrow \frac{dv}{dx}-\frac{v}{x}=-\frac{1}{x^{2}}$$

$$\text{I.F.}: e^{-\ell nx}=\frac{1}{x}$$

Sol^{n} : $\dfrac{v}{x}=-\displaystyle\int \frac{1}{x^{3}}\,dx+c$

$$=\frac{1}{2x^{2}}+c$$

$$\Rightarrow e^{-y}=\frac{1}{2x}+cx \qquad \Rightarrow 2xe^{-y}=1+2c\,x^{2}$$

Given y(1) = 0 $\Rightarrow c=\dfrac{1}{2}$

$$\Rightarrow 2x\,e^{-y}=1+x^{2}$$

$$\Rightarrow e^{y} = \frac{2x}{1+x^{2}} \Rightarrow \boxed{y = \ell n\frac{2x}{1+x^{2}}}, \; x > 0$$

When $x \to 0$ then $x \to -\infty \Rightarrow y$ is unbounded.

40. **(A,B,C,D)**

we have

$$fx(0,0) = \lim_{h\to 0}\frac{f(0+h+0)-f(0,0)}{h}$$

$$= \lim_{h\to 0}\frac{f(h,0)}{h}$$

$$= 0$$

$$\text{and sy }(0,0) = \lim_{k\to 0}\frac{f(0,0+k)-f(0,0)}{k}$$

$$= \lim_{k\to 0}\frac{f(0,k)}{k}$$

$$= 0$$

x fx + yfy = 2f

x fx - y fy = 2f

y fx + x fy = 2f

y fx - x fy = 2f

41. **1**

Sol. Let x = 2 cost, y = 2 sin t

at A : t = 0 and at B = t = $\frac{\pi}{4}$

$$\Rightarrow \bar{r}\; 2\cos t\, i + 2\sin t\, j \Rightarrow \frac{d\bar{r}}{dt} = -2\sin t\, i + 2\cos t\, j$$

$$\Rightarrow ds = \left|\frac{d\bar{r}}{dt}\right| dt = 2dt$$

$$\Rightarrow K = \int_0^{\pi/4} \frac{2\sin t}{2\cos t} 2dt = 2\int_0^{\pi/4} \tan t\, dt$$

$$= 2[\log \sec t]_0^{\pi/4}$$

$$= 2\log\sqrt{2} = \log 2 = 0.693$$

$\Rightarrow$ [K] +1 = 1

42. 9

Sol. $\nabla\phi|_{(2,-1,1)} = 8i - j - 4k$

Now the greatest value of the directional derivative of ϕ at the point

$(2,-1,1) = \left|\nabla\phi|_{(2,-1,1)}\right|$

$= \sqrt{64+1+16} = \sqrt{81} = 9$ **Ans.**

43. 1

Sol. $f_x(0,0) = \lim_{n\to 0} \frac{f(h,0) - f(0,0)}{h}$

$$= \lim_{n\to 0} \frac{\frac{h^2}{h} - 0}{h} = 1$$

$f_y(0,0) \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k}$

$= \lim_{k\to 0} \frac{0-0}{k} = 0$

$\Rightarrow f_x(0,0) + f_y(0,0) = 1$

44. 0

Sol. $\frac{\partial u}{\partial x} = f'(x+2y) + y'(x-2y)$

$$\frac{\partial^2 u}{\partial x^2} = f''(x+2y) + g''(x-2y)$$

again, $\frac{\partial u}{\partial y} = 2f'(x+2y) + 2g'(x-2y)$

$$\frac{\partial^2 u}{\partial y^2} = 4f''(x+2y) + 4g''(x-2y)$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} - 4\frac{\partial^2 u}{\partial x^2} = 0$$ **Ans.**

45. 24

Sol. Since 5 has order 6 in $\mathbb{Z}_{30}$ and 3 has order 4 in $\mathbb{Z}_{12}$

Hence $\mathbb{Z}_{12} \oplus$ < 3> is a subgroup of order 24 in $\mathbb{Z}_{30} \oplus \mathbb{Z}_{12}$.

46. 3

Sol. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & I \end{bmatrix}$

Then $A\begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}A$

$$\Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & I \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & I \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3a & a & 2b-c \\ 3d & d & 2e-f \\ 3g & g & 2h-I \end{bmatrix} \begin{bmatrix} a+b & b+e & c+f \\ 2g & 2h & 2I \\ -g & -h & -I \end{bmatrix} \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}(1)$$

When we solve (1) the we get

a = d = g = h = 0 and b = -e

$$2b - 2c - f = 0$$

$$2e - f - 2I = 0$$

$$\Rightarrow f = -(I + c)$$

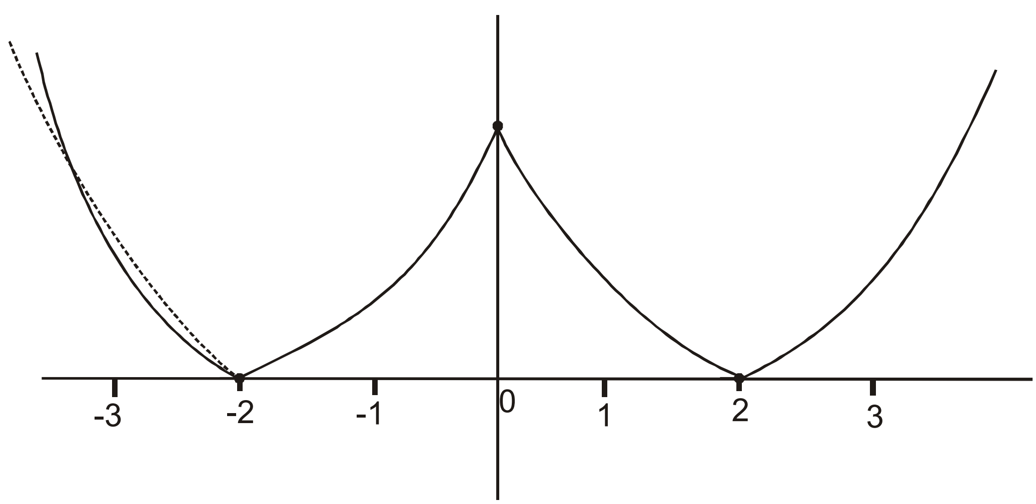
Thus $A = \begin{bmatrix} 0 & b & c \\ 0 & -b & -(I+c) \\ 0 & 0 & I \end{bmatrix}$

$\Rightarrow \dim V = 3$

47. 3

Sol. $f(x) = |(x-2)(x+2)| = \begin{cases} x^2 - 4 & , \quad x \le -2 \\ 4 - x^2 & , \quad -2 < x \le 2 \\ x^2 - 4 & , \quad x > 2 \end{cases}$

graph of f



Clearly f has 3 points of extrema.

48. - 1

Sol. As here f be a Homogeneous function of degree (-1).

So by Euler Theorem. We have

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=(-1)f(x,y)$$

$$\Rightarrow \frac{x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}}{f(x,y)}=-1 \text{ **Ans.**}$$

49. 16

Sol. $\mathrm{I}=2\int_1^4 \frac{xe^{\sin x^2}}{x^2}dx$

Let $x^2 = t \Rightarrow 2xdx = dt$

$$\Rightarrow \mathrm{I}=\int_1^{16}\frac{e^{\sin t}}{t}dt=\int_1^{16}f'(t)dt=[f(t)]_1^{16}$$

$= f(16) - f(1)$

$\Rightarrow k = 16$ **Ans.**

50. 1

Sol. We have $(2xy^2 + y)\,dx + (2y^3 - x)\,dy = 0$

$$\Rightarrow \frac{\partial M}{\partial y}=4xy+1,\ \frac{\partial N}{\partial x}=1$$

Take $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$

$$=\frac{-2(1+2xy)}{1+2xy}=-\frac{2}{y}$$

$$\Rightarrow f(x) = e^{-\log y} = \frac{1}{y^2}$$

$\Rightarrow$ f(1) = 1 **Ans.**

51. 1

Sol. Let f(x) = $x^2 + x + 1$ and $g(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$

Then f and g are inverse of each other.

So f(x) = g(x)

When f(x) = x

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow$ x = 1 **Ans.**

52. 4

Sol. Curve of intersection $z^2 = x^2 + y^2$ and $z^2 = 4y$

$x^2 + y^2 = 4y \Rightarrow x^2 + (y + z)^2 = 4$

$\therefore$ Surface Area of $z^2 = x^2 + y^2$ that projects the region R : $x^2 + (y - z)^2 = 4$ on xy-plane is

$$S = \iint \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\ dxdy$$

$$= \iint \sqrt{1 + \frac{x^2}{x2 + y^2} + \frac{y^2}{x2 + y^2}}\ dxdy$$

$$= \iint \sqrt{2}\ dxdy = \sqrt{2}\ \text{(Area of Circle)}$$

$$= \sqrt{2}\,\pi.4 = \sqrt{2}\,\pi.K$$

K = 4

53. 1.386

Sol. Now $\frac{2}{1.2.3}=\frac{1}{1.2}-\frac{1}{2.3}$

and $\frac{2}{3.4.5}=\frac{1}{3.4}-\frac{1}{4.5}$ ....So on

Thus

$$1+\frac{2}{1.2.3}+\frac{2}{3.4.5}+\frac{2}{5.6.7}+..........$$

$$=1+\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{3.4}-\frac{1}{4.5}\right)+\left(\frac{1}{5.6}-\frac{1}{6.7}\right)+.......$$

$$=1+\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{5.6}+....\right]-\left[\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+......\right]$$

$$=1+\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right).......\right]-\left[\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+.......\right]$$

$$=1+\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+.....\right]+\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-........-1\right]$$

= 1 + log 2 + log 2 - 1 = 2 log 2 = log 4 = **1.386 Ans.**

54. 4

Sol. We know $1+2+2^2+2^3+....2^{n-1}=\frac{2^n-1}{2-1}=2^n-1$

$\Rightarrow$ n^{th} term of given series is given by $\frac{2^n-1}{n!}$

$$=\frac{2^n}{n!}-\frac{1}{n!}$$

$$\Rightarrow K = \sum_{n=1}^{\infty} \frac{2^n}{n!} - \frac{1}{n!}$$

$$= \sum_{n=1}^{\infty} \frac{2^n}{n!} - \sum_{n=1}^{\infty} \frac{1}{n!}$$

$$= \left[2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + ...\right] - \left[1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\right]$$

$$= \left[1 + 2 + \frac{2^2}{2!} + ... - 1\right] - \left[1 + 1 + \frac{1}{2!} + \frac{1}{3!} .... - 1\right]$$

$= e^2 - e$

$\Rightarrow$ K = 4.67 $\Rightarrow$ [K] = 4

55. 19

Sol. Given dim(S) = 19, dim(T) = 17

Now $\dim(S \cap T)$ = dim s + dim T - dim (S + T) ________(1)

We know (S + T) is 24.

Thus minimum possible dim for $(S \cap T)$ (by (1))

= 19 + 17 - 24

= 36 - 24 = 12

Now as $S \cap T \subset S$ as well as $S \cap T \subset T$

$\Rightarrow$ Max. possible dim $(S \cap T)$ is 17.

Thus smallest possible dim for S+T is (by (1))

= 36 - 17 = 19

56. 0

Take $W^A W^T = [1 -1 1 1 -1] \begin{bmatrix} \xi^{-2} & 0 & 0 & 0 & 0 \\ 0 & \xi^{-1} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \xi & 0 \\ 0 & 0 & 0 & 0 & \xi^2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$

$$= [1 - 1 1 1 - 1]\begin{bmatrix} \xi^{-2} \\ -\xi^{-1} \\ 1 \\ \xi \\ -\xi^{2} \end{bmatrix}$$

$$= \xi^{-2} + \xi^{-1} + 1 + \xi + \xi^{2}$$

$$= \left(1 + \xi + \xi^{2} + \xi^{3} + \xi^{4}\right) / \xi^{2}$$

as ξ is fifith root of units

$\Rightarrow 1 + \xi + \xi^{2} + \xi^{3} + \xi^{4} = 0$

$\Rightarrow |W|_{A} = 0$ **Ans.**

57. 2

Sol. Given curves y = log (x + e) __________(1)

and $x = \log\frac{1}{y} \Rightarrow y = e^{-x}$ __________(2)

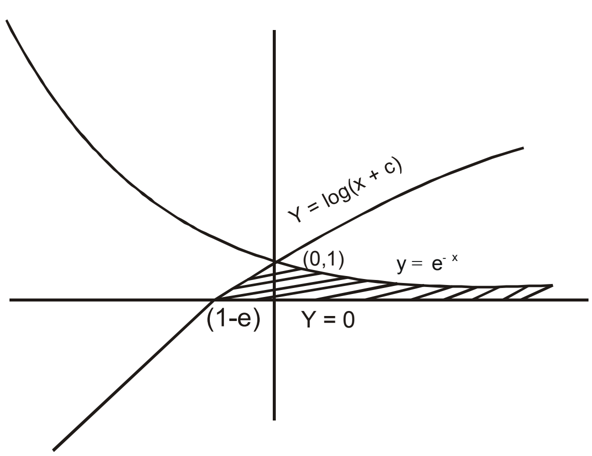
The two curves meet where log(x + e) = e^{-x}

Hence the common point is (0, 1) on y -axis.

The first curve cuts x-axis when y = 0 i.e.

log (x+ e) = 0 = log 1

$\therefore$x + e = 1 or x = 1 - e



The required area = shaded area

$$= \int_{y=0}^{1} \int_{x=e^y-e}^{\ell n\frac{1}{y}} dxdy$$

$$= \int_{0}^{1} \left(\ell n\frac{1}{y} - e^y + e \right) dy$$

$$= \left[y \log\frac{1}{y} + \int y.\frac{1}{y} dy \right]_0^1 - \left(e^y \right)_0^1 + e\left(y \right)_0^1$$

$$= \left[-y\ \ell ny + y \right]_0^1 - \left(e - 1 \right) + e$$

= 1 - e + 1 + e (as lm y lny = 0 $y \to 0$

= 2 **Ans.**

58. 0.66

We are given that, $2z = \frac{x^2}{a} + \frac{y^2}{b}$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{x}{a}, \frac{\partial z}{\partial y} = \frac{y}{b}$$

$$\Rightarrow 1 + \left(\frac{\partial z}{\partial x} \right)^2 + \left(\frac{\partial z}{\partial y} \right)^2 = 1 + \frac{x^2}{a} + \frac{y^2}{b}$$

$$\therefore S = \iint_D \sqrt{1 + \frac{x^2}{a^2} + \frac{y^2}{b^2}}\ dxdy \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (1)$$

Let x = a $\tan\theta\ \cos\theta$, y = b $\tan\theta\ \sin\theta$

Then $1 + \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 + \tan^2\theta = \sec^2\theta$

as given $\frac{x^2}{a^2} + \frac{y^2}{b^2} = K \Rightarrow \tan^2\theta = K$

$$\Rightarrow \theta = \tan^{-1}\sqrt{K}$$

The Jacobian J is written as

$$J = \frac{\partial(x,y)}{\partial(\phi,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{vmatrix} = ab\sec^2\tan\theta$$

$\therefore$ The required surface Area is given

$$S = \int_0^{2\pi} d\phi \int_0^{\theta} \sec\theta . ab\sec^2\theta\tan\theta \, d\theta$$

$$= 2\pi \, ab \int_0^{\theta} \sec^2\theta(\sec\theta\tan\theta) \, d\theta$$

$$= \frac{2\pi \, ab}{3}\left(\sec^3\theta\right)_0^{\tan-1\sqrt{k}}$$

$$= \frac{2}{3}\pi \, ab\left[\left(1+\tan^2\theta\right)^{3/2}\right]_0^{\tan-1\sqrt{k}}$$

$$= \frac{2}{3}\pi \, ab\left[(1+k)^{3/2} - 1\right]$$

$$\Rightarrow m = \frac{2}{3} = 0.66$$ **Ans.**

59. 3.771

Here $\nabla x\bar{F} = \hat{j} - 2y\hat{k}$

$$\Rightarrow \vec{k} = \int_{x=0}^{2}\int_{y=0}^{2-x}\int_{z=0}^{4-2x-2y}\left(\hat{j} - 2y\hat{k}\right)dxdy\,dz$$

$= \frac{8}{3}(j-k)$ (after calculation.

$$\Rightarrow \left|\overline{K}\right| = \frac{8}{3}\sqrt{2} = 3.771 \textbf{ Ans.}$$

60. 40

Given (2, 3,4) = (1) (2,34) (5) (6)

The no. of elements in the conjugacy class of 3-cycle (2,3,4) in the symmetric group of S_6 is

$$\frac{6!}{3!\,|\,3!\,|3!} = \frac{6!}{3.3!} = \frac{6.5.4}{3} = 40 \textbf{ Ans.}$$