BHU
MATHEMATICAL STATISTICS
SOLVED SAMPLE PAPER

* DETAILED SOLUTIONS

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BHU-MS **MATHEMATICAL STATISTICS** **FMTP**

MAX.MARKS : 360 **MARKS SCORED :** **Time : 2 Hours**

INSTRUCTIONS

Attempt all 120 questions. Each question carries 3 marks. 1 negative mark for each wrong answer.

1. Which of the following p.d.f. shows chi-square distribution with n degree of freedom?

(A) $\frac{1}{2^n \lceil n} \cdot e^{-x/2} \cdot x^{n/2-1}, 0 \le x < \infty$ (B) $\frac{1}{2^{n/2} \lceil n} e^{-x/2} \cdot x^{n-1}, 0 \le x < \infty$

(C) $\frac{1}{2^{n/2} \lceil \frac{n}{2}} e^{-x/2} \cdot x^{n-1}, 0 \le x < \infty$ (D) $\frac{1}{2^{n/2} \lceil \frac{n}{2}} e^{-x/2} \cdot x^{n/2-1}, 0 \le x < \infty$

2. Which of the following shows chi-square statistic?

(A) $\left(\frac{x-\sigma}{\mu}\right)^2$ (B) $\left(\frac{x-\mu}{\sigma^2}\right)^2$ (C) $\sum_{i=1}^{n}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2$ (D) None of these

3. Chi-square statistic is

(A) Sum of standard normal variate

(B) Sum of square of standard normal variate

(C) Square of Poisson variate

(D) None of these

4. If x_1, x_2 are independent chi-square variate with degree of freedom n_1, n_2 respectively, then which is true?

(A) $x_1 + x_2$ is chi-square with d.f. $n_1 + n_2$

(B) $x_1 - x_2$ is chi-square with d.f. $n_1 + n_2$

(C) $x_1 + x_2$ is chi-square with d.f. $n_1 - n_2$

(D) None of these

5. If x is a random variable with mean 0 and variance 1, then for any positive, number k, which is true?

(A) $P\{|x| \le k\} \le \frac{1}{k^2}$

(B) $P\{|x| \ge k\} \le \frac{1}{k^2}$

(C) $P\{|x| \ge k\} \ge \frac{1}{k^2}$

(D) $P\{|x| < k\} \ge \frac{1}{k^2}$

6. If x is a random variable with mean 1 and variance 2, then for any positive number k, which is true?

(A) $P\{|x - 1| < 2k\} \ge 1 - \frac{1}{k^2}$

(B) $P\{|x - 1| > 2k\} \ge \frac{1}{k^2}$

(C) $P\{|x - 1| \ge 2k\} \le 1 - \frac{1}{k^2}$

(D) $P\{|x - 1| \ge 2k\} \le \frac{1}{k}$

7. Let g(x) be a non-negative function of a random variable x. Then for every k > 0, we have

(A) $P\{g(x) \le k\} \le \frac{E\{g(x)\}}{k}$

(B) $P\{g(x) \ge k\} \ge \frac{E\{g(x)\}}{k}$

(C) $P\{g(x) \ge k\} \le \frac{E\{g(x)\}}{k}$

(D) None of these

8. An estimater $T_n = T(x_1, x_2, ... x_n)$ is said to be an unbiased estimater of $h(\theta)$ if

(A) $E(T_n) = \gamma(\theta), \forall \theta \in \Theta$

(B) $E(T_n) = \theta, \forall \theta$

(C) $E(T_n) = \theta^2, \forall \theta$

(D) None of these

9. If an estimator T_n of population parameter θ converges in probability to θ as n tends to infinity is said to be

(A) sufficient

(B) efficient

(C) consistent

(D) unbiased

10. The estimater $\frac{\Sigma x}{n}$ of population mean is

(A) an unbiased estimater

(B) a consistent estimater

(C) both (A) and (B) (D) Neither (A) nor (B)

11. Estimate and estimater are

(A) synonyms (B) different

(C) related to population (D) None of these

12. The estimate of θ by method of moments for the p.d.f. $f(x, \theta) = (1 + \theta^2)x$; $0 < x < 1$

(A) $\sqrt{3m_1' - 1}$ (B) $2m_1' - 1$ (C) $2m_1' + 1$ (D) m_1'

13. The estimate of θ by method of moments for t he p.d.f $f(x, \theta) = x\, e^{\theta}$; $0 < x < 1$

(A) $3m_1'$ (B) log $3m_1'$ (C) $e^{3m_1'}$ (D) - log $3m_1'$

14. Let x be the random variable which follows following distribution $f(x, \theta) = \theta \log x$; x = 1, 2. Then the estimate of θ by method of moments

(A) $\frac{m_1'}{2\log 2}$ (B) $2m_1'\log 2$ (C) $\frac{2m_1'}{2\log 2}$ (D) $\frac{2\log 2}{m_1'}$

15. Let X be the r.v which follows following distribution $f(x, \theta) = \theta^2 e^x$; x = 0, 1 then the estimate of θ by method of moments

(A) $\sqrt{\frac{m_1'}{e^{-1}}}$ (B) $\sqrt{m_1'e}$ (C) $\frac{m_1'}{e}$ (D) $m_1'e$

16. Let $x = (x_1, x_2, ...x_n)$ be the random sample and follows the distribution $f(x, \theta) = e^{-(x\theta^2 - \theta)}$, then the MLE for θ is

(A) $\frac{1}{n\bar{x}}$ (B) $\frac{1}{2\bar{x}}$ (C) $\frac{-1}{2\bar{x}}$ (D) $2n\bar{x}$

17. The random sample $x = (x_1, x_2, ...x_n)$follows the distribution $f(x, \theta) = \exp(-\theta^3 x + \theta^2)$, $x > 0$, then the MLE for θ is

(A) $\frac{3}{2}\bar{x}$ (B) $\frac{2}{3}\bar{x}$ (C) $\frac{3}{2x}$ (D) $\frac{2}{3x}$

18. If f(x) = |x|, then f'(x), where $x \neq 0$ is equal to

(A) -1 (B) 0 (C) 1 (D) $\frac{|x|}{x}$

19. A sample of size n is drawn from the distribution f(x. θ) = $\theta^2 e^{-x+1/\theta}$, x > 0, then the MLE for θ is given by

(A) $\frac{\sqrt{1-\bar{x}}}{\bar{x}}$ (B) $\frac{\sqrt{1+\bar{x}}}{\bar{x}}$ (C) $\frac{1+\sqrt{1+\bar{x}}}{\bar{x}}$ (D) $\frac{1+\sqrt{1-\bar{x}}}{\bar{x}}$

20. A sufficient statistics for θ, if (x_1, x_2, x_3,........, x_n) be a random sample form as uniform population on [0, θ] is given by

(A) $x_{(1)}$ (B) $\frac{x_{(1)} + x_{(n)}}{2}$ (C) $x_{(n)}$ (D) $\frac{x_{(n)}}{2}$

21. The sufficient statistics if (x_1, x_2, x_3,........, x_n) be a random sample from a population with p.d.f (x, θ) = $\theta x^{\theta-1}$; 0 < x < 1 θ > 0

(A) $\prod_i x_i$ (B) $\sum_i x_i$ (C) $\bar{x}$ (D) $\prod_i x_i^2$

22. Let x_1, x_2, ...,x_n be a random sample from the poisson distribution that has

p.m,f f(x, θ) = $\begin{cases} \frac{\theta^x e^{-\theta}}{x!}, & x = 0,1,2.....0 < \theta \\ 0 & \text{elsewhere} \end{cases}$, then the complete statistics for θ is given by

(A) $\sum_{i=1}^{n} x_i^2$ (B) $\sum_{i=1}^{n} x_i$ (C) $\prod_{i=1}^{n} x_i$ (D) $\prod_{i=1}^{n} x_i^2$

23. Let x_1, x_2, ...x_n be iid b(1, p) RVs. than the complete statistics for p is

(A) $\sum_{i=1}^{n} x_i^2$ (B) $\sum_{i=1}^{n} x_i$ (C) $\prod_{i=1}^{n} x_i$ (D) $\prod_{i=1}^{n} x_i^2$

24. Suppose 25% of all u.s. workers being to a labor union. What is the prob. that in a random sample of 100 u.s. workers at least 20% will belong to a labor union.

(A) 0.9875 (B) 0.7749 (C) 0.4589 (D) 0.8749

25. Assume that random samples of size 2 are drawn without replacement from the population S = {4, 7, 10} as an equiprobable space, then the mean $\mu_{\bar{x}}$ is

(A) 5 (B) 6 (C) 8 (D) 7

26. Let x be a normal random variable with mean μ and S.D. σ = 10. Find the margin of error for a 90 percent confidence interval for μ corresponding to a sample size of 12.

(A) 4.18 (B) 4.76 (C) 4.65 (D) 4.39

27. Let x be a normal random variable with unknown mean μ and S.D. σ = 3. It is desired to obtain a confidence Interval for μ with a margin or error of 1.5 based on a random sample of size is 16. What is the confidence level.

(A) .9544 (B) 9.544 (C) 90.44 (D) .09544

28. A random sample of size 10 from a normal population variable x results in the values $\bar{x} = 124$ for the sample mean and s^2 = 21 for the sample variance. Find an approximate 90 percent confidence Interval for the mean μ of x ?

(A) [121.35, 123.65] (B) [121.35, 126.65]

(C) [123.55, 126.65] (D) [122.15, 124.35]

29. Suppose 3.332 is the margin of error in a 95 percent confidence interval for the mean μ of a normal random variable x with S.D. 8.5 what is the size of the random sample ?

(A) 22 (B) 25 (C) 27 (D) 26

30. Let X_1, X_2, X_3 be a random sample of size 3 chosen from a population with probability distribution P(x = 1) = p and

P(x = 0) = 1 – p = q, 0 < p < 1. The sampling distribution f(·) of the statistic

Y = Max {X_1, X_2, X_3} is

(A) f(0) = q^3, f(1) = 1 – q^3 (B) f(0) = q, f(1) = p

(C)n f(0) = q^3, f(1) = p (D) f(0) = $p^3 + q^3$, f(1) = 1 – p^3 – q^3

31. Evaluate div $\left[\frac{f(r)\mathbf{r}}{r}\right]$, where $\vec{r}$ and r have their usual meanings.

(1) $\frac{1}{r}\frac{d}{dr}$ (r^2 f) (2) -$\frac{1}{r^2}\frac{d}{dr}$ (r f) (3) $\frac{1}{r^2}\frac{d}{dr}$ (r^2 f) (4) $\frac{1}{r^4}\frac{d}{dr}$ (r^2 f)

32. Find the curl of the vector $V = (x^2 + yz)\mathbf{i} + (y^2 + zx)\mathbf{j} + (z^2 + xy)\mathbf{k}$ at the point (1, 2, 3)

(1) 2 (2) 0 (3) 3 (4) 4

33. Evaluate curl of the vector

$\mathbf{F} = (x^2 - y^2).\ i + 2xy\ j + (y^2 - 2xy)k$

(1) $2(y + x)\ i - 2y\ j + 4y\ k$ (2) $4(y - x)\ i + y\ j - 4y\ k$

(3) $2(y - x)\ i + 2y\ j + 4y\ k$ (4) $2(y + x)\ i + 4y\ j - 2y\ k$

34. If $F = y(x + z)\ i + z(x + y)\ j + x(y + z)\ k$, find curl curl F.

(1) i + j + k (2) i - j + k (3) 2i + j - k (4) i + 2j - k

35. Find div curl F, where $F = x^2y\ i + x\ z\ j + 2yz\ k$

(1) 1 (2) 0 (3) 2 (4) 5

36. If $\mathbf{a} = a_1\ i + a_2\ j + a_3 k$ and $r = x\ i + y\ j + zk$, find curl $\{(a \times r)r^n\}$, where $r = |\ r\ |$.

(1) $(n - 2)\ r^n\ \mathbf{a} + n\ r^{n-2}\ (\mathbf{a}\ .\ \mathbf{r})\ r$ (2) $(n + 4)\ r^n\ \mathbf{a} - n\ r^{n+2}\ (\mathbf{a}\ .\ \mathbf{r})\ r$

(3) $(n + 2)\ r^n\ \mathbf{a} - n\ r^{n-2}\ (\mathbf{a}\ .\ \mathbf{r})\ r$ (4) $(n - 4)\ r^n\ \mathbf{a} + n\ r^{n-2}\ (\mathbf{a}\ .\ \mathbf{r})\ r$

37. Find out $\int_V \mathbf{B}^2\, dV$, if **B** = curl **A** and **C** = $\frac{1}{2}$ curl **B**.

(1) $\int_S (\mathbf{A} \times \mathbf{B}).\mathbf{n}\, dS + 2\int_V \mathbf{A.C}\, dV$ (2) $\int_S (\mathbf{A} \times \mathbf{B}).\mathbf{n}\, dS - 2\int_V \mathbf{A.C}\, dV$

(3) $\int_V (\mathbf{A} \times \mathbf{B}).\mathbf{n}\, dS + 4\int_S \mathbf{A.C}\, dV$ (4) $\int_S (\mathbf{A.B}).\mathbf{n}\, dS - 2\int_V \mathbf{A} \times \mathbf{C}\, dV$

38. Find out the value of $\int_S \mathbf{F} \times \mathbf{n}\, dS$

(1) $\int_V \text{curl}\, \mathbf{F}\, dV$ (2) $-\int_V \text{curl}\, \mathbf{F}\, dV$ (3) $-2\int_V \text{curl}\, \mathbf{F}\, dV$ (4) $-4\int_V \text{curl}\, \mathbf{F}\, dV$

39. Find out the value of $\int_S \phi\, \mathbf{n}\, dS$

(1) $-\int_V \text{grad}\, \phi\, dV$ (2) $2\int_V \text{grad}\, \phi\, dV$ (3) $\int_V \text{grad}\, \phi\, dV$ (4) None of these

40. Evaluate that $\nabla^2(r\,\mathbf{r})$

(1) (2)**r** (2) (2/r) $\mathbf{r}^2$ (3) $(r^3)\ \mathbf{r}$ (4) (4/r) **r**

41. If a is a constant vector, r = x i + y j + zk and $r = \sqrt{x^2 + y^2 + z^2}$, Evaluate that $\nabla\left(a.\nabla\frac{1}{r}\right)$

(1) $-\frac{a}{r^3} + \frac{3(\mathbf{a}.\mathbf{r})\mathbf{r}}{r^5}$ (2) $\frac{a}{r} - \frac{3(\mathbf{a}.\mathbf{r})\mathbf{r}}{r^5}$ (3) $\frac{a}{r^3} = \frac{(\mathbf{a}.\mathbf{r})\mathbf{r}}{2r}$ (4) $-\frac{a}{r^2} + \frac{2(\mathbf{a}.\mathbf{r}^3)\mathbf{r}}{r^3}$

42. If ρf = ∇ p, where , ρ, p, f are point functions, prove that f . ∇ × f = 0

(1) 0 (2) 1 (3) 3 (4) None of these

43. Evaluate that d ϕ

(1) $\Delta\phi$. dr (2) cosϕ . dr (3) $\nabla\phi$. dr (4) None of these

44. Evaluate that grad f (r) × **r**

(1) 2 (2) 7 (3) -1 (4) 0

45. Find grad r^m where r is the distance of any point from the origin.

(1) $mr^{m-2}\mathbf{r}$ (2) $m\,r^{m-2}$ (3) $mr^{m+2}\mathbf{r}$ (4) $m\,r^2\mathbf{r}^2$

46. Evaluate that $\nabla^2 f(r)$

(1) $f''(r) - \frac{r}{r}f'(r)$. (2) $f''(r^2) + \frac{2}{r}f(r)$.

(3) $f'(r) - \frac{r}{2^r}f''(r)$. (4) $f''(r) + \frac{2}{r}f'(r)$.

47. Evaluate that $\nabla^2\left(\frac{x}{r^2}\right)$

(1) $\frac{-2x}{x^4}$ (2) $\frac{2x}{x^2}$ (3) $\frac{x}{x^3}$ (4) $\frac{4x}{x}$

48. There are 17 balls, numbered from 1 to 17 in a bag. If a person selects one at random, what is the probability that the number printed on the ball will be an even number greater than 9 ?

(1) $\frac{13}{17}$ (2) $\frac{4}{17}$ (3) $\frac{1}{17}$ (4) $\frac{5}{17}$

49. A and B throw with three dice : if A throw 14, find B's chance of throwing a higher number .

(1) $1/12$ (2) $1/27$ (3) $26/27$ (4) $5/54$

50. There are 4 different choices available to the customer who wants to buy a transistor set. The first type costs Rs.800, the second type Rs.680, the third type Rs. 880 and the fourth type Rs. 760. The probabilities that the customer will buy these types are 1/3, 1/6, 1/4 and 1/4 respectively. The retailer of these sets gets a commission @ 20%, 12%, 25%, and 15% respectively. What is the expected commission of the retailer ?

(1) 150.43 (2) 193.4 (3) 200.43 (4)100.53

51. A food item costs Rs.30 and it can sell for Rs. 40 on the same day. Unsold item is a dead loss. It is estimated that its demand can be either 5 or 6 or 7 with probabilities 0.2, 0.7 and 0.1 respectively. If a firm store 6 items, what is its expected profit ?

(1) 3.50 (2) 4.20 (3) 5.20 (4) 6.30

52. The m.g.f. of the r.v. whose moments are $\mu r' = (r+1)2^r$.

(1) $(1-t)^{-2}$ (2) $(1-2t)^{-2}$ (3) $(1+2t)^{-2}$ (4) $(1-2t)^{2}$

53. Let the r.v. x have the distribution

$$P(X=0) = (X=2) = p \ : \ P(X=1) = 1-2p, \quad \text{for} \quad 0 \le p \le \frac{1}{2}$$

for what p is the Var (X) a maximum ?

(1) 0 (2) 9 (3) 3 (4) 1

54. Suppose that two - dimensional continuous random variable (X,Y) has joint p.d.f. given by

$$f(x,y) = \begin{cases} 6x^2y, & 0 < x < 1 \ , 0 < y < 1 \\ 0 \quad , & \text{elsewhere} \end{cases} \quad \text{, then } P(X+Y<1)$$

(1) $\frac{1}{10}$ (2) $\frac{1}{7}$ (3) $\frac{1}{9}$ (4) $\frac{8}{9}$

55. The joint p.d.f of a two - dimensional random variable (X,Y) is given by

$$f(x,y) = \begin{cases} 2: & 0 < x < 1 \ , 0 < y < x \\ 0 \quad , & \text{elsewhere} \end{cases} \quad \text{, then the marginal p.d.f. of } x = 0$$

(1) 0 (2) 1 (3) 2 (4) 3

56. The probability of bomb hitting a target is $\frac{1}{5}$. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed .

(1) 0.3557 (2) 0.3446 (3) 0.3689 (4) 0.6554

57. It is known from past experience that in a certain plant there are on the average 4 industrial accidents per month. Find the probability that in a given month there will be less than 4 accidents .(e^{-4} = 0.0183)

(1) 0.400 (2) 0.5673 (3) 0.433 (4) 0.5839

58. In a sample of 1000 items, the mean weight is 45 kg. with standard deviation of 15 kgs . Assuming the normality of the distribution, the number of items weighing between 40 and 60 kg.

(1) 471 (2) 591 (3) 480 (4) 552

59. The wage distribution of the workers in a factory is normal with mean Rs. 400 and S.D. Rs. 50. If the wages of 40 workers be less than Rs. 350, what is the total number of workers in the factory ?

(1) 150 (2) 250 (3) 100 (4) 200

60. A pair of unbiased dice is rolled together till a sum of either 5 or 7 is obtained. The probability that 5 comes before 7 is

(1) 2/5 (2) 3/5 (3) 4/5 (4) none of these

61. If every element of group $(G, \bullet)$ is its own inverse then. $(G, \bullet)$ is

(A) Abelian group (B) Non Abelian group

(C) Cyclic group (D) None of these

62. Set S = {–2, –1, 1, 2} with respect to multiplication, is

(A) a group (B) not a group

(C) Monoid (D) Semi group

63. In a group G $b^{-1}a^{-1}ba = e$ then G is

(A) Abelian (B) Non Abelian

(C) Sub group (D) None of these

64. If H is subgroup of finite group G and order of H and G are respectively m and n then

(A) m/n (B) n/m

(C) $m \times n$ (D) None of these

65. If H_1 and H_2 are two subgroup of G then, following is also subgroup of G:-

(A) $H_1 \cap H_2$ (B) $H_1 \cup H_2$

(C) H_1H_2 (D) None of these

66. If H is non void subset of group G and $a \in H, b \in H \Rightarrow ab^{-1} \in H,$ then H is

(A) Abelian group (B) Subgroup

(C) Cyclic Subgroup (D) None of these

67. Which of the following function T from V_2(R) into V_2(R) is not a linear transformation?

(A) T (x, y) = (y, x) (B) T(x, y) = (x + y, x)

(C) T(x, y) = (1 + x, y) (D) T (x, y) = (x – y, y – x)

68. The linear transformation T :F^3 (C) ® F^3(C) defined as

$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2 - x_3, -x_1 - 2x_2)$.

Then the null space of T is

(A) $\{(0,0,-2)\ (-1,0,2)\}$ (B) {(0, 0, 0)}

(C) {(1, 0, 0) (0, 0, 0)} (D) None of these

69. Let V(F) be the set of all polynomials in x over F of degree ≤ 5. If linear transformation D : V ® V is defined by

D [f(x)] = f′ (x). Where f′ (x) is the derivative of f(x). The matrix of D in the basis $\{1, x^2, x^3, x^4\}$ is –

(A) $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ (D) None of the above

70. Let $(\alpha_1, \alpha_2, \alpha_3)$ be an ordered basis for R^3 where $\alpha_1 = (1, 0, -1)$, α_2 = (1, 1, 1), vector (1, 2, 2) is –

(A) (1, 2, 1) (B) (0, 2, – 1)

(C) (1 , 1, 1) (D) (0, 2 , 1)

71. The linear transformation $T : R^2 \to R^2$ such that T (1, 0) = (1, 1) and T (0, 1) = (-1, 2) is defined by –

(A) $T(x_1, x_2) = (x_1 - 2x_2, x_1 + x_2)$ (B) $T(x_1, x_2) = (2x_1 - x_2, x_1 - x_2)$

(C) $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$ (D) $T(x_1, x_2) = (x_1 - 2x_2, x_1 + 2x_2)$

72. The linear transformation $T : R^2 \to R^2$ such that T(2, 3) = (12, 15) and T (1, 0) = (0, 0) is defined by –

(A) $T(x_1, x_2) = (x_1 - x_2, 4x_2)$ (B) $T(x_1, x_2) = (4x_2, 5x_2)$

(C) $T(x_1, x_2) = (4x_1, 5x_2)$ (D) $T(x_1, x_2) = (4x_2, 5x_1)$

73. If T is a linear operator on R^2 defined by T (x, y) = (x – y, y) then T^2 (x, y)

(A) (x^2, y^2) (B) (2x – y, 2y)

(C) (x – 2y, y) (D) None of these

74. Suppose that X is normally distributed with a mean of 50 and a standard deviation of 6. The value of c, such that Pr(50 - c < X < 50 + c) = 0.95 is closest to:

(A) 11.76 (B) 1.96 (C) 1.65 (D) 9.87

75. Suppose that pulse rates of people in a certain population are normally distribute(D) If 70% of people have pulse rates greater than 65 beats per minute, and 10% of people have pulse rates of more than 80 beats per minute, then the mean and the standard deviation of pulse rate in this population are closest to:

(A) = 73.3, = 8.3 (B) = 54.6, = 19.8

(C) = 69.4, = 8.3 (D) = 75.4, = 8.3

76. Out of the numbers 1 to 100, one is selected at random. The probability that it is divisible by 7 or 8 .

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

77. In a random arrangement of the letters of the word 'COMMERCE' . The probability that all the vowels come together

(A) $\frac{3}{28}$ (B) $\frac{25}{28}$ (C) $\frac{27}{28}$ (D) $\frac{1}{28}$

78. A, B and c are three mutually exclusive and exhaustive events, then P(B), if

$$\frac{1}{3}(C) = \frac{1}{2}P(A) = P(B)$$

(A) $\frac{1}{6}$ (B) $\frac{1}{7}$ (C) $\frac{1}{8}$ (D) $\frac{6}{7}$

79. Out of the 1000 persons born only 800 reach the age of 10, and out of every 1000 who reach the age of 10,850 reach the age of 40 . Out of every thousand who reach the age of 40, 25 die in one year. what is the probability that a person would attain the age of 41 years ?

(A) 0.607 (B) 0.650 (C) 0.690 (D) 0.663

80. A candidate is selected for interview for three posts. For the first post there are 5 candidates, for the second there are 8 and for the third there are 7. What are the chances for his getting at least one post ?

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

81. A and B throw one die for a prize of Rs. 11, which is to be won by the player who first throw 6. If A has the first throw, what are their respective expectations ?

(A) Rs. 6 and Rs. 5 (B) Rs. 5 and Rs. 7

(C) Rs. 7 and Rs. 5 (D) Rs. 6 and Rs. 7

82. If $\Sigma x_i = 225, \Sigma y_i = 189, \Sigma(x_i - 22)^2 = 92.5, \Sigma(y_i - 19)^2 = 40.4, \Sigma(x_i - 22)(y_i - 19) = 47$ and n = 10 then the value of $r_{X,Y}$ is

(A) 0.48 (B) 0.86 (C) – 0.48 (D) 0.8

83. For a negatively correlated distribution if b_{YX} and b_{XY} are coefficients of regression then coefficient of correlation is given by

(A) – b_{YX}

(B) – b_{XY} b_{YX}

(C) $-\sqrt{b_{YX}b_{XY}}$

(D) $\sqrt{b_{YX}b_{XY}}$

84. If the two regression coefficients are positive then

(A) $1/b_{YX} + 1/b_{XY} > 2/r$

(B) $1/b_{YX} + 1/b_{XY} < 2/r$

(C) $1/b_{YX} + 1/b_{XY} < r/2$

(D) None of these

85. Given that $\sum XY$ = 120, $\sum Y = 432$, $\sum XY = 4992$, $\sum X^2 = 1392$, $\sum Y^2 = 18,\ 252$, N = 12.

Find out the regression co-efficients.

(A) 3.5 and 0.249

(B) 2.5 and 0.249

(C) 2.5 and 0.449

(D) 3.5 and 0.449

86. Given that $\sum XY$ = 120, $\sum Y = 432$, $\sum XY = 4992$, $\sum X^2 = 1392$, $\sum Y^2 = 18,\ 252$, N = 12.

The regression equation of Y on X is

(A) Y = 3X + 1

(B) X = 0 .249 Y + 1.036

(C) X = 0.249 Y + 2.9

(D) Y = 3.5 X + 1

87. An unbiased die is rolled twice. Let A denote the event that an even number appears first time and B denote the event that an odd number appears second time. Then A and B

(A) mutually exclusive

(B) independent and mutually exclusive

(C) independent

(D) None of these

88. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14, The probability that neither A nor B occurs is

(A) 0.39 (B) 0.25 (C) 0.11 (D) None of these

89. A student appears for tests I, II, and III . The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the students passing in tests I, II, III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2 then possible values of p and q.

(A) p = q = 1 (B) p = q = 1/2 (C) p = 1, q = 0 (D) p = 1, q = 1/2

90. Evaluate $\frac{d}{dt}\left[\mathbf{a}\,\frac{d\mathbf{a}}{dt}\,\frac{d^2\mathbf{a}}{dt^2}\right]$

(A) $-\left[a\,\frac{da}{dt}\,\frac{d^3a}{dt^3}\right]$ (B) $\left[a\,\frac{da}{dt}\,\frac{d^4a}{dt^4}\right]$ (C) $\left[a\,\frac{da}{dt}\,\frac{d^3a}{dt^3}\right]$ (D) $\left[a\,\frac{d^2a}{dt^2}\,\frac{d^3a}{dt^3}\right]$

91. If f (x, y, z) = $x^2 y + y^2 x + z^2$, find ∇ f at the point (1, 1, 1)

(A) 3 i - 3j + 2k (B) 3 i + 3j - 2k

(C) 3 i - 3j - 2k (D) 3 i + 3j + 2k

92. If r = xi + yj + zk and f = $|\,r\,|^3$, then find $\nabla |\,r\,|^3$

(A) 4r **r** (B) 2r **r** (C) 3r **r** (D) 5r **r**

93. Find ∇f if f = $(x^2 + y^2 + z^2)\ e^{-\sqrt{(x^2+y^2+z^2)}}$

(A) $(2 - r)\ e^{r}$ **r** (B) $(2 - r)\ e^{-r}$ **r**

(C) $(2 + r)\ e^{-r}$ **r** (D) $(2 + r)\ e^{r}$ **r**

94. Evaluate ∇e^{r^2} , where $r^2 = x^2 + y^2 + z^2$

(A) $2e^{r^2}$ r (B) $2e^{r^{-2}}$ r (C) e^{r^2} r (D) $e^{r^{-2}}$ r

95. IF $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, find the value of grad {log $|\vec{r}|$ }

(A) $\frac{\vec{r}}{|\vec{r}|^2}$ (B) $\frac{-\vec{r}}{|\vec{r}|^2}$ (C) $\frac{\vec{r}}{|\vec{r}|^3}$ (D) None of these

96. Find ∇ r^{-3}, where r = | r | = |xi + yj +zk|.

(A) $-2r^{-5}$ **r** (B) $2r^{-5}$ **r** (C) $-3r^{-5}$ **r** (D) $3r^{-5}$ **r**

97. If f (x, y) = log $\sqrt{(x^2 + y^2)}$ Find grad f .

(A) $\frac{r + (k.r)k}{\{r - (k.r)k\}.\{r - (k.r)k\}}$ (B) $\frac{r - (k.r)k}{\{r + (k.r)k\}.\{r + (k.r)k\}}$

(C) $\frac{r - (k.r)k}{\{r - (k.r)k\}.\{r - (k.r)k\}}$ (D) $\frac{r + (k.r)k}{\{r + (k.r)k\}.\{r + (k.r)k\}}$

98. Evaluate $a \cdot \nabla r$

(A) a (B) -a (C) 0 (D) None of these

99. If $F = (x^3 + y^3 + z^3 - 3xyz)$ find $\nabla \cdot F$

(A) 6 (x + y + z) (B) 3 (x + y + z)

(C) 3 (x - y - z) (D) 6 (x - y - z)

100. If r = xi + yj + zk, then find div r

(A) 2 (B) 3 (C) 1 (D) None of these

101. If $F = xy^2\, i + 2x^2 yz\, j - 3yz^2 k$, find div F at the point (1, –1, 1).

(A) 8 (B) 9 (C) 4 (D) 3

102. Evaluate div $\hat{r}$ or div (**r** / r), where **r** and r have their usual meanings.

(A) $\frac{2}{r}$ (B) $\frac{-2}{r}$ (C) $\frac{1}{r}$ (D) $\frac{-1}{r}$

103. Find the total work done in moving a particle in a field of force given by F = 3xyi – 5zj + 10xk along the curve C given by x = t, $y = t^2 + 1$, $z = t^3$ from t = 0 to t = 2.

(A) 74 (B) 75 (C) 78 (D) 65

104. Find $\int_S \mathbf{F.n}\, dS$, where F = $\nabla\phi$ and $\nabla^2\phi = -4\pi\rho$.

(A) $4\pi\int_V \rho\, dV$ (B) – $2\pi\int_V \rho\, dV$ (C) – $2\pi\int_V \rho\, dV$ (D) $-4\pi\int_V \rho\, dV$

105. Evaluate $\int_S \phi \mathbf{n}\, dS$

(A) $\int_V -\nabla\phi dV$ (B) $\int_V \nabla\phi dV$ (C) $\int_V 2\nabla\phi dV$ (D) None of these

106. Evaluate $\int_V \mathbf{A}.\nabla\phi\, dV$

(A) $\int_S \mathbf{A}\, \phi\, dS - \int_V \phi \text{ div } \mathbf{A}\, dV$ (B) $\int_S \mathbf{n}\, \phi\, dS - \int_V \phi \text{ div } \mathbf{A}\, dV$

(C) $\int_S \mathbf{A.n}\, \phi\, dS - \int_V \phi \text{ div } \mathbf{A}\, dV$ (D) None of these

107. The volue of Evaluate $I = \iint_{\Re} (x - y) dA$, where $\Re$ is the region above the x-axis bounded by $y^2 = 3x$ and $y^2 = 4 - x$

(A) $24\sqrt{3} + \frac{9}{2}$ (B) $\frac{24}{5}\sqrt{3} + \frac{9}{2}$

(C) $\frac{24}{5}\sqrt{3}-\frac{9}{2}$ (D) None of these

108. The complementary function of equation $\frac{d^2y}{dx^2}$ + y = xe^{2x} is

(A) $c_1\cos x + c_2\sin x$ (B) $(c_1 + c_2x)\cos x$

(C) $(c_1 + c_2x)\sin x$ (D) $c_1\cos 2x + c_2\sin 2x$

109. The particular integral of equation $\frac{d^2y}{dx^2}$ – y = coshx is

(A) sinhx (B) coshx (C) $\frac{1}{2}$xsinhx (D) $\frac{1}{2}$xcoshx

110. $\ell\frac{d^2\theta}{dt^2}$ + gθ = 0 has a solution when t = 0, θ = 80, $\frac{d\theta}{dt}$ = 0

(A) $\theta_o\sin\left(\sqrt{\frac{g}{\ell}}\right)t$ (B) $\theta_o\cos\left(\sqrt{\frac{g}{\ell}}\right)t$

(C) $c_1\cos\sqrt{\frac{g}{\ell}}\,t + c_2\sin\sqrt{\frac{g}{\ell}}\,t$ (D) None of these

111. Solution of equation $\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{i}{LC}=0$ where R^2C = 4L and R, C, L is constant.

(A) $(C_1+C_2t)e^{\frac{-R}{2L}}$ (B) $(C_1+C_2t)e^{\frac{R}{2L}}$ (C) $C_1e^{\frac{R}{2L}}$ (D) $C_1e^{\frac{-R}{2L}}$

112. The homogeneous linear differential equation $y^{(n)} + P_o y^{(n-1)}$ + ------ + $P_n y$ = 0 has general solution of the coefficient $P_o(x)$ ------ $P_n(x)$ on same interval I are

(A) Continuous (B) Discontinuous

(C) Discontinuous and differentiable (D) None of these

113. For equation $\frac{d^2y}{dx^2}$ + 4y = tan2x, solving by variation of parameters. The value of wronskion is

(A) 1 (B) 2 (C) 3 (D) 4

114. Solving by variation of parameter y'' – 2y' + y = e^xlogx, the value of wronskion w is

(A) e^{2x} (B) 2 (C) e^{-2x} (D) None of these

115. E and F be two independent events such that P(E) < P (F). The probability that both E and F happen is 1/12 and the probability that neither E nor F happen is 1/12. Then

(A) P (E) = 1/3, P (F) = 1/2 (B) P(E) = 1/2, P(F) = 2/3

(C) P (E) = 2/3, P (F) = 3/4 (D) P(E) = 1/4, P (F) = 1/3

116. A Man is known toss speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six. is

(A) 3/8 (B) 1/5

(C) 3/4 (D) none of these

117. If (1 + 3p)/3, (1 - p)/4 and (1 - 2p)/2 are the probabilities of three mutually exclusive, events then the set of all values of p is

(A) 1/3 $\leq$ p 1/2 (B) 1/4 $\leq$ 9 $\leq$ 1/3

(C) - 1 $\leq$ p $\leq$ 1/5 (D) - 2 $\leq$ 9 $\leq$ 1/3

118. The value of wronskion w(x, x^2, x^3) is

(A) $2x^4$ (B) $2x^2$ (C) $2x^3$ (D) None of these

119. If m is a natural such that $m \leq 5$ then the probability that the quadratic equation

$x^2 + mx + \frac{1}{2} + \frac{m}{2} = 0$ has real roots is

(A) 1/5 (B) 2/3 (C) 3/5 (D) 1/5

120. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in random order till both the faulty machines are identified. Then the probability that the only two tests are needed is

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

ANSWER KEY BHU MS FMTP

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	D	C	B	A	B	A	C	A	C	C	B	A	B	A	A	B	D	D	D	C
Question	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	A	B	B	D	D	B	A	B	B	D	C	B	C	A	B	C	A	B	C	D
Question	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	A	A	C	D	A	D	A	B	D	A	C	B	D	A	A	B	C	A	B	A
Question	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Answer	A	B	A	A	A	B	C	B	C	B	D	B	C	A	C	B	A	A	D	C
Question	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Answer	A	D	C	A	A	D	C	A	C	C	D	C	B	A	A	C	C	A	A	B
Question	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Answer	B	A	A	D	B	C	C	A	C	B	A	A	B	A	D	A	A	C	C	B

HINTS AND SOLUTIONS

1.(D) For chi-square distribution with n. d. f. p.d.f. is given by $f(x) = \frac{1}{2^{\frac{n}{2}}\Gamma\frac{n}{2}} . e^{-x/2} . x^{n/2-1}, \; 0 \le x < \infty$

2.(C) By definition of chi-square distribution

3.(B) By the definintion of chi-square distribution

4.(A) By additive proberty of chi-square distribution

5.(B) Using chebychev's inequality

$$P(|x - \mu| \ge k\sigma) \le \frac{1}{k^2}$$

here $\mu = 0, \quad \sigma = 1$

$$\therefore \quad P(|x| \ge k) \le \frac{1}{k^2}$$

6.(A) By another form of chebys hev's inequality

$$P(|x - \mu| < k\sigma) \ge 1 - \frac{1}{k^2}$$

Here $\mu = 1, \sigma = 2$

$$P\{|x - 1| < 2k\} \ge 1 - \frac{1}{k^2}$$

7.(C) By generalised form of bienayme -chebyshev's inequality

$$P\{g(x) \geq k\} \leq \frac{E\{g(x)\}}{k}$$

8.(A) By definition of unbiasedness

9.(C) It is definition of consistent estimator

10.(C) Obviously

11.(B) Estimate and estimator are different

12.(A) $f(x,\theta) = (1+\theta^2)x : \quad 0 < x < 1$

$$r_1' = \int_0^1 x.(1+\theta^2)x = (1+\theta^2)\int_0^1 x^2\,dx = (1+\theta^2)\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{(1+\theta^2)}{3}$$

$$\Rightarrow \theta^2 = 3r_1'-1 \Rightarrow \theta = \sqrt{3r_1'-1}$$

Replace r_1' by m_1' , $\therefore \hat{\theta} = \sqrt{3m_1'-1}$

13.(B) $f(x, \theta) = xe^\circ$, $0 < x < 1$

$$\therefore \quad r'_1 = E(x) = \int_0^1 x.xe^\theta dx = e^\theta \int_0^1 x^2\,dx = \frac{e^\theta}{3}$$

$$\therefore \quad 3r_1' = e^\theta \Rightarrow \theta = \log 3r_1'$$

$$\therefore \quad \hat{\theta} = \log 3m_1'$$

14.(A) $f(x, \theta) = \theta \log x \quad ; \quad x = 1, 2$

$$\therefore \quad \mu_1' = \sum_{x=1}^{2} x.\theta\log x = \theta\sum_{x=1}^{2} x\log x = \theta[0+2\log 2]$$

$$\Rightarrow \quad \mu_1' = 2\theta\log 2 \Rightarrow \theta = \frac{\mu_1'}{2\log 2}$$

$$\therefore \quad \hat{\theta} = \frac{m'_1}{2\log 2}$$

15.(A) $f(x, \theta) = \theta^2 e^x, \quad x = 0, 1$

$$\therefore \quad \mu_1' = \sum_{x=0}^{1} x.\theta^2 e^x = \theta^2 \sum_{x=0}^{1} x e^x = \theta^2[0+e]$$

$\Rightarrow \qquad \theta^2 = \frac{\mu_1'}{e} \Rightarrow \theta = \sqrt{\frac{\mu_1'}{e}}$

$\therefore \qquad \hat{\theta} = \sqrt{\frac{m_1'}{e}}$

16.(B) f(x, θ) = $e^{-(x,\theta^2-\theta)}$

The likelihood function

$$L = \prod_i f(x_i, \theta) = \prod_i e^{-(x_i\theta^2-\theta)}$$

$$\Rightarrow \qquad L = e^{-\theta^2(\Sigma x_i)+n\theta}$$

$$\Rightarrow \qquad \ell n\ L = \{-\theta^2(\Sigma x_i) + (n\theta)\}\ \ell ne$$

$$\Rightarrow \qquad \ell n\ L = -\theta^2(\Sigma x_i) + (n\theta)$$

Now, $\frac{\partial}{\partial\theta}\ell n\, L = -2\theta(\Sigma x_i) + n = 0$

$$\Rightarrow 2\theta(\Sigma x_i) = n \Rightarrow \theta = \frac{n}{2(\Sigma x_i)} = \frac{1}{2\left(\frac{\Sigma x_i}{n}\right)}$$

$\therefore \qquad \theta = \frac{1}{2\bar{x}}$

Thus, the MLE for $\theta = \hat{\theta} = \frac{1}{2\bar{x}}$

Also, we may check $\left(\frac{\partial^2}{\partial\theta^2}\ell nL\right) < 0$

$\therefore \qquad \hat{\theta} = \frac{1}{2\bar{x}}$ gives the maximum value.

17.(D) f(x, θ) = exp. {-(θ³x + θ²)} , x > 0

$$L = \prod_i f(x_i, \theta) = \prod_i \exp.\{-(\theta^3 x + \theta^2)\},\ x > 0$$

$$\Rightarrow \qquad L = e^{-\theta^3\Sigma x_i + n\theta^2}, x > 0$$

$$\Rightarrow \qquad \ell n\ L = -\theta^3\Sigma x_i + n\theta^2$$

$$\Rightarrow \quad \frac{\partial}{\partial\theta}\ell n\, L = -3\theta^2\Sigma x_i + 2n\theta = 0$$

$$\Rightarrow \quad \theta\left[-3\theta\Sigma x_i + 2n\right] = 0$$

$$\Rightarrow \quad \theta = \frac{2}{3}\frac{n}{\Sigma x_i} = \frac{2}{3}\frac{1}{\bar{x}} = \frac{2}{3\bar{x}}$$

18.(D) $\because\ f(x) = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$

$$\because \quad f'(x) = \begin{cases} 1,\ x > 0,\ \text{ie},\ \dfrac{|x|}{x},\ x > 0 \\ -1,\ x < 0,\ \text{ie},\ \dfrac{|x|}{x},\ x < 0 \end{cases}$$

$$= \frac{|x|}{x},\ x \ne 0$$

19.(D) $f(x, \theta) = \theta^2 e^{-x\theta + 1/\theta}$

$$L = \prod_i f(x_i, \theta) = \prod_i \theta^2 e^{-x+1/\theta} = \left(\theta^2\right)^n e^{-(\Sigma x_i)\theta + n/\theta}$$

$$\therefore \quad \ln L = 2n(\ell n\theta) - \theta\Sigma x_i + \frac{n}{\theta}$$

$$\Rightarrow \frac{\partial}{\partial\theta}\ln L = \frac{2n}{\theta} - (\Sigma x_i) - \frac{n}{\theta^2} = 0$$

$$\Rightarrow \ 2n\theta - (\Sigma x_i)\theta^2 - n = 0$$

$$\Rightarrow \ (\Sigma x_i)\theta^2 - 2n\theta + n = 0$$

$$\therefore \quad \theta = \frac{2n \pm \sqrt{4n^2 - 4n\,\Sigma x_i}}{2\,\Sigma x_i} = \frac{2n \pm 2n\sqrt{1-\bar{x}}}{2\Sigma x_i} = \frac{1 \pm \sqrt{1-\bar{x}}}{\bar{x}}$$

$\therefore$ The MLE for $\theta = \dfrac{1 \pm \sqrt{1-\bar{x}}}{\bar{x}}$

20.(C) Given $f_\theta(x_i) = \begin{cases} \dfrac{1}{\theta}, & 0 \le x_i \le \theta \\ 0 & \text{, otherwise} \end{cases}$

Let $K(a,b)=\begin{cases}1, & \text{if } a\le b\\ 0, & \text{if } a>b\end{cases}$, then $f_\theta(x_i)=\dfrac{k(0,x_i)k(x_i,\theta)}{\theta}$

$$L=\prod_{i=1}^{n} f_\theta(x_i)=\prod_{i=1}^{n}\left[\frac{k(0,x_i)k(x_i,\theta)}{\theta}\right]=\frac{k\left(0,\min\limits_{1\le i\le n} x_i\right)\cdot k\left(\max\limits_{1\le i\le n} x_i,\ \theta\right)}{\theta^n}=g_\theta\left[t(x)\right]h(x)$$

where $g_\theta\left[t(x)\right]=\dfrac{k\{t(x),\theta\}}{\theta^n}$, $t(x)=\underset{1\le i\le n}{\text{Max}}\, x_i$ and $h(x)=k\left(0,\min\limits_{1\le i\le n} x_i\right)$

$\therefore$ $T=\underset{1\le i\le n}{\text{Max}}\, x_i=x_{(n)}$, is sufficient for θ.

21.(A) The likelihood function

$$L(x,\theta)=\prod_{i=1}^{n} f(x_i,\theta)=\theta^n\prod_{i=1}^{n}\left(x_i^{\theta-1}\right)$$

$$=\theta^n\left(\prod_{i=1}^{n} x_i\right)^{\theta}=\frac{1}{\left(\prod_i x_i\right)}=g(t_1,\theta).h(x,x_2,\ldots x_n)$$

$\therefore$ $t_1=\prod_i x_i$ is sufficient for θ.

22.(B) Let $y_1=\sum_{i=1}^{n} x_i$, we know that y_1 is sufficient for θ.

Let $E[u(y_1)]=0$ for $\theta>0$

$$\Rightarrow \quad \sum_{y_1=0}^{\infty} u(y_1)\frac{(n\theta)^{y_1}e^{-n\theta}}{y_1!}=0$$

$$\Rightarrow \quad e^{-n\theta}\left[u(0)+u(1)\frac{n\theta}{1!}+u(2)\frac{(n\theta)^2}{2!}+\ldots\right]=0$$

$$\Rightarrow \quad u(0)=0,\ nu(1)=0,\ \frac{n^2u(2)}{2}=0\ldots.$$

$$\Rightarrow \quad 0=u(0)=u(1)=u(2)=\ldots..$$

$\therefore$ y_1 is complete.

23.(B) Let $T=\sum_{i=1}^{n} x_i$. we know T is sufficient for p.

Now, let E[g(T)] = 0

$$\Rightarrow \sum_{t=0}^{n} g(t)\, {}^{n}c_t p^t (1-p)^{n-t} = 0 \qquad \forall\, p \in (0,1)$$

$$\Rightarrow (1-p)^n \sum_{t=0}^{n} g(t)\, {}^{n}c_t \left(\frac{p}{1-p}\right)^t = 0 \qquad \forall\, p \in (0,1)$$

This is a polynomial in $\frac{p}{1-p}$ Hence the coefficient must vanish and it follows that g(t) = 0 for t = 0,1 , 2, ...n

$\therefore \quad T = \sum_{i=1}^{n} x_i$ is complete for p.

24.(D) The sample size n = 100 > 30 and the total no. of u.s. workers is much larger than 100. therefore, the sample proportion p of workers that belong to a labour union can be modded as a nomal random variable with mean p = 0.25 and standard deviation

$$\sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.25 \times 0.75}{100}} \approx 0.0433. \text{ Then}$$

$z = \frac{\hat{p} - 0.25}{0.0433}$ is standard normal variate

$$p(\hat{p} \geq 0.2) = p\left(\frac{\hat{p} - 0.25}{0.0433} \geq \frac{0.2 - 0.25}{0.0433}\right)$$

$$= P(z \geq -1.15)$$

$$= P(z \leq 1.15)$$

$$= 0.8749$$

25.(D)

$\bar{x}$	:	5.5	7	8.5
$p(\bar{x})$	:	1/3	1/3	1/3

$$r_{\bar{x}} = E(\bar{x}) = (5.5)\frac{1}{3} + 7 + (8.5) \cdot \frac{1}{3} = \frac{21}{3} = 7$$

26.(B) $\because \quad E = \frac{z^* \sigma}{\sqrt{n}}$, where $\sigma = 10, n = 12$

we find $p(-z^* \leq Z \leq z^*) = 0.9$ for $z^* = 1.65$

$E = \frac{1.65 \times 10}{\sqrt{12}} \approx 4.76$

27.(A) $\because$ $E = \frac{z^* \sigma}{\sqrt{n}}$ we have $1.5 = \frac{z^* \times 3}{\sqrt{16}} = \frac{3z^*}{4}$

So $z^* = \frac{4 \times 1.5}{3} = 2$

we find $p(-2 \le z \le 2) = 2p(0 \le z \le 2) = 2(0.4772) = 0.9544$

28.(B) $E = \frac{t^* s}{\sqrt{n}}$ where $S = \sqrt{21}, n = 10$

$P(-t^* \le t \le t^*) = 0.90$

$P(0 \le t \le t^*) = 0.45$ then $t^* = 1.83$

$\therefore$ $E = \frac{1.83\sqrt{21}}{\sqrt{10}} \approx 2.65$

$\therefore$ $[124 - 2.65, 124 + 2.65] = [121.35, 126.65]$

29.(B) $3.332 = \frac{1.96 \times 8.5}{\sqrt{n}}$

$\therefore \sqrt{n} = \frac{1.96 \times 8.5}{3.332}$

$x = 25$

30.(D) Let X_1, X_2, X_3 be a random sampling of size 3 chosen from a population with probability distribution P(x = 1) = p and P(x = 0) = 1 – p = q, 0 < p < 1

Then the sampling distribution f(·) of the statistic

$Y = \text{Max}\{X_1, X_2, X_3\}$ is

$f(0) = p^3 + q^3$

and $f(1) = 1 - p^3 - q^3$

31.(3) f(r) **r** = f(r) (x**i** + y**j** + z**k**)

$$\text{div}\left[\frac{f(r)r}{r}\right] = \frac{\partial}{\partial x}\left\{\frac{f(r)x}{r}\right\} + \frac{\partial}{\partial y}\left\{\frac{f(r)y}{r}\right\} + \frac{\partial}{\partial z}\left\{\frac{f(r)z}{r}\right\} \quad \text{....... (i)}$$

Now $\frac{\partial}{\partial x}\left\{\frac{f(r)x}{r}\right\} = x\,\frac{\partial}{\partial x}\left\{\frac{f(r)}{r}\right\} + \frac{f(r)}{r}\frac{\partial}{\partial x}(x)$

or $\frac{\partial}{\partial x}\left[\frac{f(r)x}{r}\right] = x\left\{\frac{1}{r}f'(x)\frac{\partial r}{\partial x} - \frac{1}{r^2}f(r)\frac{\partial r}{\partial x}\right\} + \frac{f(r)}{r} \quad = \left\{\frac{x}{r}f'(r) - \frac{x}{r^2}f(r)\right\}\frac{\partial r}{\partial x} + \frac{f(r)}{r}$

$$= \left\{\frac{x}{r}f'(r) - \frac{x}{r^2}f(r)\right\} . \frac{x}{r} + \frac{f(r)}{r},$$

$\because \quad \frac{\partial r}{\partial x} = \frac{x}{r}$

$$= \frac{x^2}{r^2}\ f(r) - \frac{x^2}{r^3}\ f(r) + \frac{f(r)}{r}$$

Similarly we can get

$\frac{\partial}{\partial y}\left[\frac{f(r)y}{r}\right] = \frac{y^2}{r^2}\ f(r) - \frac{y^2}{r^3}\ f(r) + \frac{f(r)}{r}$ and $\frac{\partial}{\partial z}\left[\frac{f(r)z}{r}\right] = \frac{z^2}{r^2}\ f(r) - \frac{z^2}{r^3}\ f(r) + \frac{f(r)}{r}$

Substituting these values in (i) get

div $\left[\frac{f(r)\mathbf{r}}{r}\right] = \frac{x^2+y^2+z^2}{r^2}\ f(r) - \frac{x^2+y^2+z^2}{r^2}\ f(r) + \frac{3f(r)}{r}$

$= f(r) - \frac{1}{r}\ f(r) + \frac{3}{r}f(r), \quad x^2 + y^2 + z^2 = r^2 \quad = f(r) + \frac{2}{r}\ f(r)$, where $f'(r) = \frac{d}{dr}\ f(r)$

$= \frac{1}{r^2}\ [r^2 f(x) + 2\ r\ f(r)] = \frac{1}{r^2}\left[r^2\frac{d}{dr}f(r) + 2rf(r)\right] \quad = \frac{1}{r^2}\ \frac{d}{dr}\ [r^2\ f\ (r)]$ Hence proved

32.(2) Curl V = ∇ x V

$= \left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right)$ x $[(x^2 + yz)\mathbf{i} + (y^2 + zx)\mathbf{j} + (z^2 + xy)\mathbf{k}] = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+yz & y^2+xz & z^2+xy \end{vmatrix}$

$= \mathbf{i}\left[\frac{\partial}{\partial y}(z^2+xy) - \frac{\partial}{\partial z}(y^2+zx)\right] + \mathbf{j}\left[\frac{\partial}{\partial z}(x^2+yz) - \frac{\partial}{\partial x}(z^2+xy)\right] + \mathbf{k}\left[\frac{\partial}{\partial x}(y^2+zx) - \frac{\partial}{\partial y}(x^2+yz)\right]$

$= \mathbf{i}\ (x - x) + \mathbf{j}\ (y - y) + \mathbf{k}\ (z - z) = 0 \qquad \forall x, y, z$

Curl V at (1, 2, 3) is 0

33.(3) **Curl F** = $\nabla \times F$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times \left[(x^2 - y^2)i + 2xyj + (y^2 - 2xy)k\right] = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 - y^2 & 2xy & y^2 - 2xy \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y^2 - 2xy) - \frac{\partial}{\partial z}(2xy)\right] - j\left[\frac{\partial}{\partial x}(y^2 - 2xy) - \frac{\partial}{\partial z}(x^2 - y^2)\right] + k\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial y}(x^2 - y^2)\right]$$

$= i\ (2y - 2x) - j\ (-2y) + k\ (2y + 2y) = 2\ (y - x)\ i + 2y\ j + 4y\ k$

34.(1) curl F = – y i – zj – x k

$\therefore$ curl curl F = $\nabla \times$ (curl F) = $\nabla \times [-y\ i - z\ j - x\ k]$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z & -x \end{vmatrix} = \sum i\left[\frac{\partial}{\partial y}(-x) - \frac{\partial}{\partial z}(-z)\right]$$

$= i\ [0 + 1] - j\ [-1\ \ -0] + k\ [0 + 1] \quad = i + j + k$

35.(2) Given F = x^2 y i + x z j + 2 y z k

$$\therefore \text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & xz & 2yz \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(2yz) - \frac{\partial}{\partial z}(xz)\right] - j\left[\frac{\partial}{\partial x}(2yz) - \frac{\partial}{\partial z}(x^2y)\right] + k\left[\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(x^2y)\right] = i\ [2z - x] - j\ [0] + k\ [z - x^2]$$

$\therefore$ div curl F = ∇ . $[(2z - x)\ i + 0\ j + (z - x^2)\ k\ ]$

$$= \frac{\partial}{\partial x}(2z - x) + \frac{\partial}{\partial z}(z - x^2) \qquad = -1 + 1 = 0$$

36.(3) $a \times r = (a_1 i + a_2 j + a_3 k) \times (x i + y j + z k) = a_1 y\, i \times j + a_1 z\, i \times k + a_2 x\, j \times i + a_2 z\, j \times k + a_3 x\, k \times i + a_3 y\, k \times j$

$\because i \times i = 0$ etc

$= a_1 y k - a_1 z j + a_2 x(-k) + a_2 z i + a_3 x j + a_3 y(-i),$

$\because i \times j = k = -j \times i$

$= (a_2 z - a_3 y) i + (a_3 x - a_1 z) j + (a_1 y - a_2 x) k$

$(a \times r) r^n = [(a_2 z - a_3 y) r^n] i + [(a_3 x - a_1 z) r^n] j + [(a_1 y - a_2 x) r^n] k$

$= b_1 i + b_2 j + b_3 k$

where $b_1 = (a_2 z - a_3 y) r^n$ etc

$\therefore \text{curl } (a \times r) r^n = \nabla \times [(a \times r) r^n]$

$$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \times (b_1 i + b_2 j + b_3 k) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \sum i\left[\frac{\partial}{\partial y}(b_3) - \frac{\partial}{\partial z}(b_2)\right] = \sum i\left[\frac{\partial}{\partial y}\{(a_1 y - a_2 x) r^n\} - \frac{\partial}{\partial z}\{(a_3 x - a_1 z) r^n\}\right]$$

$$= \sum_i \left[a_1 r^n + (a_1 y - a_2 x) n r^{n-1}\frac{\partial r}{\partial y} + a_1 r^n - (a_3 x - a_1 z) n r^{n-1}\frac{\partial r}{\partial z}\right]$$

$= 2r^n \sum a_1 i + n r^{n-1} \sum i\left[(a_1 y - a_2 x)\frac{y}{r} - (a_3 x - a_1 z)\frac{z}{r}\right] = 2r^n a + n r^{n-2} \sum i\, [a_1(y^2 + z^2) - x(a_2 y + a_3 z)]$

$= 2r^n a + n r^{n-2} \sum i [a_1(x^2 + y^2 + z^2) - x(a_1 x + a_2 y + a_3 z)] = 2 r^n a + n r^{n-2} \sum i [a_1 r^2 - x(\mathbf{a} \cdot \mathbf{r})]$

$= 2 r^n a + n r^n \sum (a_1 i) - n r^{n-2} (\mathbf{a} \cdot \mathbf{r}) \sum (x i) = 2r^n \mathbf{a} + n r^n \mathbf{a} - n r^{n-2} (\mathbf{a} \cdot \mathbf{r})\, r$

$= (n + 2) r^n \mathbf{a} - n r^{n-2} (\mathbf{a} \cdot \mathbf{r})\, r$

37.(1) Putting $\mathbf{F} = \mathbf{A} \times \mathbf{B}$ in Gauss theorem we have

$\int_S (\mathbf{A}\times\mathbf{B}).\mathbf{n}\ dS = \int_V \text{div}\ (\mathbf{A}\times\mathbf{B})\ dV$ = $\int_V (\mathbf{B}.\ \text{curl}\ \mathbf{A} - \mathbf{A}.\text{curl}\ \mathbf{B})dV$

∵ curl A = B and curl **B** = 2**C** (given)

= $\int_V \mathbf{B}^2\ dV - 2\int_V\ \mathbf{A.C}\ dV$ or $\int_V \mathbf{B}^2\ dV = \int_S (\mathbf{A}\times\mathbf{B}).\ \mathbf{n}\ dS + 2\int_V \mathbf{A.C}\ dV$ Answer.

38.(2) We known Gauss divergence theorem is

$\int_S \mathbf{A.n}\ dS = \int_V \text{div}\ \mathbf{A}\ dV$

Put **A** = **a** × **F**, where **a** is any arbitrary constant vector.

Then $\int_S$ (**a** × **F**) . **n** dS = $\int_V \text{div}\ (\mathbf{a}\times\mathbf{F})dV$ or $\int_S (\mathbf{a.F}\times\mathbf{n})\ dS = \int_V \nabla.(\mathbf{a}\times\mathbf{F})dV$

∵ **a**. **F** × **n** = a × **F** . **n**

= – $\int_V (\mathbf{a.}\ \nabla\times\mathbf{F})dV$, $\because \mathbf{a.b}\times c = -\mathbf{b.a}\times c$ or $\mathbf{a.}\int_S (\mathbf{F}\times\mathbf{n})dS = -\mathbf{a.}\int_V \nabla\times\mathbf{F}\ dV$

∵ **a** is a constant vector

or **a**. $\left[\int_S \mathbf{F}\times\mathbf{n}\ dS + \int_V \nabla\times\mathbf{F}\,dV\right] = 0$ or $\int_S \mathbf{F}\times\mathbf{n}\ dS + \int_V \nabla\times\mathbf{F}\,dV = 0$

∵ **a** is an arbitrary vector

or $\int_S \mathbf{F}\times\mathbf{n}\ dS = -\int_V \nabla\times\mathbf{F}\,dV$ or $\int_S \mathbf{n}\times\mathbf{F}\ dS = \int_V \nabla\times\mathbf{F}\,dV$ Answer.

39.(3). We know Gauss divergence theorem is

$\int_S \mathbf{F.n}\ dS = \int_V \text{div}\ \mathbf{F}\ dV$

Put **F** = **a**ϕ, where **a** is any arbitrary constant vector.

Then $\int_S \mathbf{a.}\phi\mathbf{n}\ dS = \int_V \text{div}(\mathbf{a}\phi)\ dV$ or $\int_S \mathbf{a.}\phi\mathbf{n}\ dS = \int_V \nabla.(\mathbf{a}\phi)\ dV$

or **a**. $\int_S \phi\mathbf{n}\ dS = \mathbf{a.}\int_V \nabla.\phi\ dV$

∵ **a** is constant vector.

or **a**. $\left[\int_S \phi\mathbf{n}\ dS - \int_V \nabla.\phi\ dV\right] = 0$ or $\int_S \phi\mathbf{n}\ dS - \int_V \nabla.\phi\ dV = 0$,

∵ **a** is arbitrary

or $\int_S \phi \mathbf{n}\, dS = \int_V \nabla.\phi\, dV$ Answer.

40.(4) $\nabla^2(r\,\mathbf{r}) = \frac{\partial^2}{\partial x^2}(r\,\mathbf{r}) + \frac{\partial^2}{\partial y^2}(r\,\mathbf{r}) + \frac{\partial^2}{\partial z^2}(r\,\mathbf{r})$

$= \sum \frac{\partial^2}{\partial x^2}(r\mathbf{r}) = \sum \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}(r\mathbf{r})\right] = \sum \frac{\partial}{\partial x}\left[\frac{\partial r}{\partial x}\mathbf{r} + r\frac{\partial r}{\partial x}\right] = \sum \frac{\partial}{\partial x}\left[\frac{x}{r}\mathbf{r} + r\,i\right],$

$\because \frac{\partial r}{\partial x} = \frac{\partial}{\partial x}$ (x i + y j + z k) = i, $\frac{\partial r}{\partial x} = \frac{x}{\partial r}$

$= \sum \left[\left\{\frac{r}{r} + \frac{x}{r}\frac{\partial r}{\partial x} - \frac{x}{r^2}\mathbf{r}\frac{\partial r}{\partial x}\right\} + \mathbf{i}\frac{\partial r}{\partial x}\right] = \sum \left[\left\{\frac{\mathbf{r}}{r} + \frac{x}{r}(i) - \frac{\mathbf{r}x}{r^2}\left(\frac{x}{r}\right)\right\} + i\frac{x}{r}\right],$

$\because \frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial \mathbf{r}}{\partial x} = i$

$= \frac{3r}{r} + \frac{1}{r}(xi + yj + zk) - \frac{\mathbf{r}}{r^3}(x^2 + y^2 + z^2) + \frac{1}{r}(xi + yj + z) = \frac{3\mathbf{r}}{r} + \frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r} + \frac{\mathbf{r}}{r},$

$\because x^2 + y^2 + z^2 = r^2$, x i + y j + z k = r

$= \frac{4}{r}\mathbf{r}$

41.(1) $\nabla(1/r) = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\left[\frac{1}{\sqrt{(x^2 + y^2 + z^2)}}\right]$

$= \sum i[-\frac{1}{2}(x^2 + y^2 + z^2)^{-3/2}.2x] = -\sum i[x(x^2 + y^2 + z^2)^{-3/2}] = -\sum i[x(r^2)^{-3/2}] = -r^{-3}\sum(xi) = -r^{-3}\,\mathbf{r}$

$\therefore$ a . ∇ (1 / r) = a . ($-r^{-3}$ r) = $-r^{-3}$ (**a** . **r**)

$= -r^{-3}(a_1x + a_2y + a_3z)$, If $a = a_1 i + a_2 j + a_3 k$

$\therefore \nabla[a . \nabla(1/r)]$

$= \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)\left[-\frac{a_1}{r^3}x - \frac{a_2}{r^3}y - \frac{a_3}{r^3}z\right]$

$\nabla\left[a.\nabla\left(\frac{1}{r}\right)\right] = -\sum i\left[\frac{\partial}{\partial x}\left(\frac{xa_1}{r^3}\right) + \frac{\partial}{\partial x}\left(\frac{a_2y}{r^3}\right) + \frac{\partial}{\partial x}\left(\frac{a_3z}{r^3}\right)\right]$ .... (i)

$\left[\frac{a_1}{r^3} - \frac{3}{r^4} x a_1 \frac{\partial r}{\partial x}\right]$, where $\frac{\partial r}{\partial x} = \frac{x}{r}$

$$\frac{\partial}{\partial x}\left[\frac{x a_1}{r^3}\right] = \left[\frac{a_1}{r^3} - \frac{3a_1 x^2}{r^5}\right] \quad .... \text{(ii)}$$

$$\frac{\partial}{\partial x}\left[\frac{a_2 y}{r^3}\right] = a_2 y\left[-\frac{3}{r^4}\frac{\partial r}{\partial x}\right] = -\frac{3a_2 y}{r^4}\left(\frac{x}{r}\right) = -\frac{3a_2 xy}{r^5} \quad .... \text{(iii)}$$

and $\frac{\partial}{\partial x}\left[\frac{a_3 z}{r^4}\right] = a_3 z\left[-\frac{3}{r^4}\frac{\partial r}{\partial x}\right] = -\frac{3a_3 xz}{r^5}$ .... (iv)

$\therefore$ From (i), (ii), (iii) and (iv) we get

$$\nabla[a \cdot \nabla(1/r)] = -\sum i\left[\frac{a_1}{r^3} - \frac{3a_1 x^2}{r^5} - \frac{3a_2 xy}{r^5} - \frac{3a_3 zx}{r^5}\right]$$

$$= -\sum i\left[\frac{1}{r^3}a_1 - \frac{3x}{r^5}(a_1 x + a_2 y + a_z)\right] = -\frac{1}{r^3}\sum i a_1 + \frac{3}{r^5}(a_1 x + a_2 y + a_z)\sum x i = -\frac{a}{r^3} + \frac{3}{r^5}(\mathbf{a} \cdot \mathbf{r})\, r$$

42.(1) Given $f = \frac{1}{\rho}\nabla p$

$\therefore \nabla \times f \; \nabla\left[\frac{1}{\rho}\nabla p\right]$ or $\nabla \times f = \frac{1}{\rho}(\nabla \times \nabla p) + \nabla\left(\frac{1}{\rho}\right) \times \nabla p$

$\because \nabla(\phi V) = \phi(\nabla \times V) + (\nabla\phi) \times V$

$= \frac{1}{\rho}(0) + \nabla\left(\frac{1}{\rho}\right) \times \nabla p,$

$\because$ curl $\nabla \phi = 0$, sec or $(\nabla \times f) = \nabla\left(\frac{1}{\rho}\right) \times \nabla p$

$\therefore \mathbf{f} \cdot (\nabla \times f) = \left[\frac{1}{\rho}\nabla p\right] \cdot \left\{\nabla\left(\frac{1}{\rho}\right) \times \nabla p\right\}$

$$= \frac{1}{\rho}\left\{\nabla p . \nabla\left(\frac{1}{\rho}\right) \times \nabla p\right\} = \frac{1}{\rho}\left[\nabla p, \nabla\left(\frac{1}{\rho}\right) \times \nabla p\right] = \frac{1}{\rho}(0)$$

$\because$ [abc] = 0, if a = c

= 0

43.(3) We know $d\,\phi = \frac{\partial \phi}{\phi x}dx + \frac{\partial \phi}{\phi y}dy + \frac{\partial \phi}{\phi z}dz$

$= \left(\mathbf{i}\frac{\partial \phi}{\phi x} + \mathbf{j}\frac{\partial \phi}{\phi y} + \mathbf{k}\frac{\partial \phi}{\phi z}\right)$ **.** ($\mathbf{i}$ dx + $\mathbf{j}$ dy + $\mathbf{k}$ dz) = ($\nabla \phi$) **.** (d $\mathbf{r}$), $\because$ r = x i + y j + z k

44.(4) Let $\phi = f(r) = f\left(x^2 + y^2 + z^2\right)^{1/2}$

$$\frac{\partial \phi}{\partial x} = f'(r)\frac{y}{r}, \frac{\partial \phi}{\partial x} = r'(r)\frac{x}{r}$$

Similarly. $\frac{\partial \phi}{\partial y} = f'(r)\frac{y}{r}, \frac{\partial \phi}{\partial z} = r'(r)\frac{z}{r}$

$\therefore$ grad f (r) $= \sum \mathbf{i}\frac{\partial \phi}{\partial x} \quad = \frac{1}{r}f'(r).(\mathbf{i}x + \mathbf{j}y + \mathbf{k}z) = \frac{f'(r)}{r}\mathbf{r}$

$\therefore$ grad f (r) × **r** $= \frac{f'(r)}{r}\mathbf{r} \times \mathbf{r} = 0$.

45.(1) We have

$$\phi(x, y, z) = r^m = \left(x^2 + y^2 + z^2\right)^{\frac{m}{2}} \qquad \frac{\partial \phi}{\partial x} = mx\left(x^2 + y^2 + z^2\right)^{\frac{m}{2}-1} = mxr^{m-2}$$

$$\frac{\partial \phi}{\partial y} = myr^{m-2}, \qquad \frac{\partial \phi}{\partial z} = mzr^{m-2}$$

Then grad $r^m = \sum \mathbf{i}\frac{\partial \phi}{\partial x} \quad = mr^{m-2}\sum \mathbf{i}x \quad = mr^{m-2}\mathbf{r},$

Where, **r,** iw the position vector of any point. We can also write.

grad $r^m = mr^{m-1}\left(\frac{\mathbf{r}}{r}\right),$ where **r**/r is the unit vector joining the origin to any point.

46.(4) Let $\phi = f(r);$

$\therefore \quad \nabla^2 f(r) = \nabla^2 \phi = \sum \frac{\partial^2 \phi}{\partial x^2}. \qquad \frac{\partial \phi}{\partial x} = f'(r)\frac{\partial r}{\partial x} = \frac{x}{r}f'(r)$

$\therefore \quad \frac{\partial^2 \phi}{\partial x^2} = \frac{r\left\{1.f'(r) + xf''(r).\frac{x}{r}\right\} - xf'(r).\frac{x}{r}}{r^2} \qquad = \frac{1}{r}f'(r) + \frac{x^2}{r^2}f''(r) - \frac{x^2}{r^3}f'(r)$

$$\therefore \quad \sum \frac{\partial^2 \phi}{\partial x^2} = \frac{3}{r}f'(r) + \frac{\sum x^2}{r^2}f''(r) - \frac{\sum x^2}{r^3}f'(r) \quad = \frac{3}{r}f'(r) + f''(r) - \frac{1}{r}f'(r) \quad = f''(r) + \frac{2}{r}f'(r).$$

47.(1) $$\frac{\partial^2}{\partial x^2}\left(\frac{x}{r^2}\right) = \frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}\left(\frac{x}{r^2}\right)\right] \quad = \frac{\partial}{\partial x}\left[1.\frac{1}{r^2} - \frac{2x}{r^3}.\frac{x}{r}\right] \quad \left[\because \frac{\partial r}{\partial x} = \frac{x}{r}\right]$$

$$= \frac{\partial}{\partial x}\left[\frac{1}{r^2} - \frac{2x^2}{r^4}\right] = \left[-\frac{2}{r^3}.\frac{x}{r} - \frac{4x}{r^4} + \frac{8x^2}{r^5}.\frac{x}{r}\right] \quad = \left[-\frac{2}{r^4}x - \frac{4x}{r^4} + \frac{8x^3}{r^6}\right] = -\frac{6x}{r^4} + \frac{8x^3}{r^6} = -2x\left[\frac{3}{r^4} - \frac{4x^2}{r^6}\right] \quad ....(1)$$

$$\frac{\partial^2}{\partial y^2}\left(\frac{x}{r^2}\right) = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial y}\left(\frac{x}{r^2}\right)\right] \quad = \frac{\partial}{\partial y}\left[-\frac{2x}{r^3}.\frac{\partial r}{\partial y}\right] \quad = \frac{\partial}{\partial y}\left[-\frac{2x}{r^3}.\frac{y}{r}\right] = \frac{\partial}{\partial y}\left[-\frac{2xy}{r^4}\right]$$

$$= -2x\left[\frac{1}{r^4} - \frac{4y}{r^5}.\frac{y}{r}\right] = -2x\left[\frac{1}{r^4} - \frac{4y^2}{r^6}\right] \quad ....(2)$$

Similarly, $$\frac{\partial^2}{\partial z^2}\left(\frac{x}{r^2}\right) = -2x\left[\frac{1}{r^4} - \frac{4z^2}{r^6}\right] \quad ....(3)$$

$$\therefore \quad \nabla^2\left(\frac{x}{r^2}\right) = \sum \frac{\partial^2}{\partial x^2}\left(\frac{x}{r^2}\right) \quad = -2x\left[\frac{3}{r^4} + \frac{1}{r^4} + \frac{1}{r^4} - \frac{4}{r^6}\left(x^2 + y^2 + z^2\right)\right]$$ by (1), (2) and (3)

$$= -2x\left[\frac{5}{r^4} - \frac{4}{r^4}\right] = -\frac{2x}{r^4}.$$

48. (2) The total no. of ways of drawing a ball out of 17 balls = ${}^{17}C_1 = 17$. Since even numbers greater than 9 in 1 to 17 numbers are 10, 12, 14 and 16, the favourable number of ways of selecting a ball are thus 4 .

$\therefore$ The required probability $= \frac{4}{17}$

49.(4) To throw higher number than A, B must throw either 15 or 16 or 17 or 18 .

Now a throw amounting to 18 must be made up of (6,6,6) which can occur in one way : 17 can be made up of (6, 6, 5) which, can occur in 3 ways : 16 may be made up of (6,6,4) and (6, 5, 5), each of which arrangements can occur in 3 ways : 15 can be made up of (6,4,5) or (6,3,6) or (5, 5, 5) which can occur in 3!, 3 and 1 way respectively .

$\therefore$ The no. of favourable cases = 1 + 3 + 3 + 3 + 6 + 3 + 1 = 20 And the exhaustive number of cases 6^3 or 216

Hence, the required probability = $\frac{20}{216}=\frac{5}{54}$.

50.(1) If we let x stand for the random variable representing the commission of the retailer, then x may assume the values 160 (i.e. 20% of 800) : 81.60 (i.e. 12% of 680) : 220 (i.e. 25% of 880) and 114 (15 % of 760) respectively.

The probability distribution for x is

x :	160	81.60	220	114
p(x) :	1/3	1/6	1/4	1/4

The expected commission of the retailer is :

$$E(x)=160\times\frac{1}{3}+81.60\times\frac{1}{6}+220\times\frac{1}{4}+114\times\frac{1}{4} \quad = \text{Rs. } 150.43$$

51.(3) Let the r.v.x represents the profit of the firm.

Then the probability digit fox X is

Profit X	:	5 × 10 – 1 × 30 = 20	6 × 10 = 60	6 × 10 = 60
p (x)	:	0.2	0.7	0.1

$$E(x)=20\times0.2+60\times0.7+60\times0.1 \;=\; \text{Rs. } 52$$

Hence the expected profit to the firm is Rs. 5.20 .

52.(2) The m.g.f. is given by

$$M_x(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\,\mu_r{}'=\sum_{r=0}^{\infty}\frac{t^r}{r!}(r+1)!\;\;2^r=\sum_{r=0}^{\infty}(r+1)(2t)^r$$

$$=1+2(2t)+3(2t)^2+4(2t)^3+..........=(1-2t)^{-2}$$

53.(4) Here the r.v. X takes the value 0, 1, and 2 with respective probability p, 1 – 2p and p, $0\le p\le,\frac{1}{2}$, Thus

$$E(X)=0\times p+1\times(1-2p)+2\times p=1,\quad E\left(X^2\right)=0\times p+1^2\times(1-2p)+2^2\times p = 1+2p$$

$$\therefore \qquad Var(X)=E\left(X^2\right)-\left[E(x)\right]^2=2p \quad ; \;\; 0\le p\le\frac{1}{2}$$

Obviously , for $0\le p\le\frac{1}{2}$, $Var(x)$ is maximum when $p=\frac{1}{2}$.

and $[Var(X)]_{max}$ = 2 × 1/2 = 1

54.(1) $P(X+Y<1)=\int_0^1\int_0^{1-x}6x^2y\,dx\,dy=\int_0^1 6x^2\left|\frac{y^2}{2}\right|_0^{1-x}dx \quad =\int_0^1 3x^2(1-x)^2\,dx=\frac{1}{10}$

55.(1) $f(x)=\begin{cases}\int_{-\infty}^{\infty} f(x,y)\,dy = \int_{\sigma}^{x} 2dy=2x, & 0<x<1\\ 0, & \text{elsewhere}\end{cases} \Rightarrow \quad f(x=0) = 0$

56.(2) In the usual notations : n = Number of bombs = 6 , p = Probability of a bomb hitting the target = $\frac{1}{5}$ so that $q=\frac{4}{5}$

Now P(r) = Probability that out of 6 bombs, r hit the bridge =

$^6C_r\left(\frac{1}{5}\right)^r\left(\frac{4}{5}\right)^{6-r}$; $r=0,1,2,3,.........,6.$

Since the bridge is destroyed if at least two of the bombs hit it, the probability that the bridge is destroyed is given by :

p(2) + p(3) + p(4) + p(4) + p(5) + p(6) = 1 – [p(0) + p (1)]

$$=1-\left[\left(\frac{4}{5}\right)^6+6\times\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^5\right].$$

$$=1-\frac{1}{5^6}\left[4^6+6\times4^5\right]=1-\frac{2048}{3125}=0.3446$$

57.(3) Let X denote the number of accidents occurring in the plant every month. Then X has a Poisson distribution with parameter :

m = Average number of accidents per month = 4

Then by Poisson probability law : $P(X=r)=p(r)=\frac{e^{-4}4^r}{r!};\quad r=0,1,2,.........$

Required probability = $P(X\le4)=p(0)+p(1)+p(2)+p(3)$

$= e^{-4}(1+4+8+10.67)=0.433$

58.(1) Let X be a normal variate with mean (μ) = 45 and s.d. (σ) = 15 . We have to calculate $P(40\le X\le60)$.

Now $P(40\le X\le60) = P\left(\frac{40-45}{15}\le Z\le\frac{60-65}{15}\right)$

$= P(-0.03\le Z\le1)$

$= P(-0.03 \le Z \le 0) + P(0 \le Z \le 1)$

$= P(0 \le Z \le 0.33) + P(0 \le Z \le 1)$ (Due to symmetry)

= 0.1293 + 0.3413 = 0.4796

Hence, the expected number of items weighing between 40 and 60 kgs. 100 × 0.4706 $\cong 471$.

59.(2) Let the variable X denotes the wages (in Rs.) of the workers. Then we are given :

$X \sim N(\mu, \sigma^2)$, where $\mu = 400$, $\sigma = 50$.

The probability that the wages of worker is less than Rs. 40 is given by P (X < 40) .

When $X = 40 ; Z = \frac{X - \mu}{\sigma} = \frac{350 - 400}{50} = -1$

$\therefore \quad P(X < 350) = P(Z < 1) = P(X > 1) = 0.5 - P(0 \le Z \le 1) = 0.16$

Now 16% of the workers = 40

Hence total number of workers = $\frac{40 \times 100}{16}$ = 250 .

60.(1)

Let A denote the event that the a sum of 5 occurs, B the event that a sum of 7 occurs and C the event that neither a sum of 5 nor a sum of 7 occurs.

P (1) = $\frac{4}{36} = \frac{1}{9}$, P (2) = $\frac{6}{36} = \frac{1}{6}$ and P (3) = $\frac{26}{36} = \frac{13}{18}$. Thus,

P (A occurs before B)

= P [A or $(C \cap A)$ or $(C \cap C \cap A)$ or ...]

= P (1) + P $(C \cap A)$ + P $(C \cap C \cap A)$ + ...

= P (1) + P (3) P (1) + $P(3)^2$ P (1) + ...

$$= \frac{1}{9} + \left(\frac{13}{18}\right) \times \frac{1}{9} \times \left(\frac{13}{18}\right)^2 \frac{1}{9} + ...$$

$$= \frac{1/9}{1 - 13/18} = \frac{2}{5}$$ [Sum of an infinite G.P]

67.(C) Let a = $((a_1, a_2), \beta = (b_1, b_2) \in V_2(R)$

Then T (A) = $(1 + a_1, a_2)$ and T(B) = $(1 + b_1, b_2)$

Also let a, b $\in R$, then $a\alpha + b\beta \in V_2(R)$

$\therefore \quad T(a\alpha + b\beta) = T[a(a_1, a_2) + b(b_1, b_2)]$

= T $(aa_1 + bb_1, aa_2 + bb_2)$

= $(1 + aa_1 + bb_1, aa_2 + bb_2)$

$\neq$ aT(A) + bT(B)

Hence, T is not a linear transformation from $V_2(R)$ in $V_2(R)$.

68.(B) (a, b, c) $\in$ null space of t

$\Leftrightarrow$ T (a, b, c) = (0, 0, 0)

$\Leftrightarrow$ (a-b + 2c, 2a + b – c, –a – 2b) = (0, 0, 0)

$\Leftrightarrow$ a –b + 2c = 0, 2a + b – , –a –2b + 0c = 0 ...(1)

Coefficient matrix A of equation (1) is

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & -2 & 0 \end{bmatrix} \Rightarrow |A| = -9 \neq 0$$

$\therefore$ Rank (A) = 3

Hence, the equations (1) have no linearly independent solutions. So a_1 = 0, b = 0, c = 0 is the only solution of the equations (1)

$\therefore$ (0, 0, 0) is the only vector which belongs to null space of T.

69.(C) We have D (1) = 0 = $0.1 + 0.x + 0.x^2 + 0.x^3 + 0.x^4$

f" (x) = D (x) = 0 = $0.1 + 0.x + 0.x^2 + 0.x^3 + 0.x^4$

f" (x^2) = D (x^2) = 2 = $2.1 + 0.x + 0.x^2 + 0.x^3 + 0.x^4$

f" (x^3) = D (x^3) = 6x = $0.1 + 6.x + 0.x^2 + 0.x^3 + 0.x^4$

f" (x^4) = D (x^4) = $12x^2$ = $0.1 + 0.x + 12.x^2 + 0.x^3 + 0.x^4$

$$\therefore \quad \text{Required matrix} = \begin{bmatrix} 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

70.(B) Let p, q, r be scalars such that

$(1, 2, 2) = p\,\alpha_1 + q\alpha_2 + r\alpha_3$

$\Rightarrow$ (1, 2, 2) = p (1, 0, –1) + q (1, 1, 1) + r (1, 0 ,0)

$\Rightarrow$ (1, 2, 2) = (p + q + r, q, - p + q)

$\Rightarrow$ p + q + r = 1, q = 2 and –p + q = 2

Solving these, we get

p = 0, q = 2 and r = – 1.

71.(D) We have

T (1,0) = (1 –0, 1 + 2.0) = (1, 1)

and T (0, 1) = (0 –1, 0 + 2.1) = (-1,2).

72.(B) We have

T (2,3) = (4 × 3, 5 × 3) = (12, 15)

and T (1, 0) = (4 × 0,5 × 0) = (0, 0).

73.(C) $\because$ T (x, y) = (x – y, y)

$\therefore$ T^2 (x, y) = T (x – y, y)

= (x – y – y, y)

= (x – 2y, y)

76.(B) The sample space in the random experiment is : S = {1,2,3,......99,100}

The number of elements in the sample space ,i.e., the exhaustive number of outcomes is given by n (S) = 100 .

The event E : number chosen is divisible by 7' has the sample points given by :

$$E = \{7,14,21,28,..........,98\} \text{ and } n(E) = \frac{98}{7} = 14$$

Similarly the event F : the number chosen is divisible by 8' has the sample points given by :

$F = \{8, 16, 24, ..., 96\}$ and $n(F) = \frac{96}{8} = 12$

Also $E \cup F = \{56\}$ and $n(E \cap F) = 1$

Hence, the required probability is :

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{14}{100} + \frac{12}{100} - \frac{1}{100} = \frac{25}{100} = \frac{1}{4}$.

77.(A) The number of distinct permutations of the letters of the word 'COMMERCE', is given by $\frac{8!}{2!2!2!}$, because it contain 8 letters of which C,M and E are repeated twice and the remaining letters are all different .The word 'COMMERCE' contains 3 vowels, viz O, E,E of which these 3 vowels come together,we regard them as tied together, forming only one letter so that total number of letters in COMMERCE may be taken as 8-2 = 6, out of which 2 are C's, 2 are M's and rest distinct and, therefore, their number of arrangements is given by $\frac{6!}{2!2!}$

Further, the three vowels OEE two of which are identical and real distinct can be arranged themselves in 3!/2! ways. Hence, the total number of arrangements favourable to getting all vowels together is : $\frac{6!}{2!2!} \times \frac{3!}{2!}$

$\therefore \quad \text{Required probability} = \frac{6!3!}{2!2!2!} + \frac{8!}{2!2!2!}$

$= \frac{6!3!}{8!} = \frac{3 \times 2}{8 \times 7} = \frac{3}{28}$

78.(A) Since A, B and C are there mutually exclusive and exhaustive events,

$P(B), \text{if } \frac{1}{3}P(C) = \frac{1}{2}P(A) = P(B)$.

or $\quad 2P(B) + P(B) + 3P(B) = 1 \quad [\because P(A) = 2P(B), P(C) = 3P(B)]$

$\therefore \quad 6P(B) = 1 \qquad$ Hence $P(B) = \frac{1}{6}$

79.(D) Let us define the events :

E : The person reaches the age of 10 .

F : The person who reaches the age of 10, also reaches the age of 40 .

G : The person who reaches the age of 40, attains the age of 41 .

We are given : $P(E) = \frac{800}{1000}, P(F) = \frac{850}{1000}, P(\bar{G}) = \frac{25}{1000}$

Required probability = $P(E \cap F \cap G) = P(E) \times P(F) \times P(G)$

[Since E, F and G are independent events.]

= $\frac{800}{1000} \times \frac{850}{1000} \times \left(1 - \frac{25}{1000}\right) = 0.663$

80.(C) Let E, F, and G denote the events that the candidate is selected for the first, second and third post respectively. Since the selection of each candidate is equally likely, we have

$P(E) = \frac{1}{5} \qquad$ or $\qquad P(\bar{E}) = \frac{4}{5}$

$P(F) = \frac{1}{8} \qquad$ or $\qquad P(\bar{F}) = \frac{7}{8}$

$P(G) = \frac{1}{7} \qquad$ or $\qquad P(\bar{G}) = \frac{6}{7}$

The required probability that the candidate is selected for at least one post is :

$$P(E \cup F \cup G) = 1 - P(\bar{E} \cap \bar{F} \cap \bar{G})$$

$$= 1 - P(\bar{E}) \times P(\bar{F}) \times P(\bar{G})$$ [Since the events E, F and G are independent .]

$$= 1 - \frac{4}{5} \times \frac{7}{8} \times \frac{6}{7} = \frac{2}{5}\ .$$

81.(A) If A is to win, 6 must be thrown on 1st, 3rd,5th,............. throws and A's chance of winning is the sum of these probabilities. Similarly if B is to win, 6 must be throw on 2nd, 4th, 6th,.......... throws .

Let E_1 and E_2 denotes the events that A and B gets 6 respectively .

$$\therefore \qquad P(E_1) = \frac{1}{6} = P(E_2) \ \Rightarrow \ P(\bar{E}_1) = \frac{5}{6} = P(\bar{E}_2)$$

Probability of A's winning in first throw is $p_1 = P(E_1) = \frac{1}{6}$

Probability of A's winning in third throw is

$$p_2 = P(\bar{E}_1 \cap \bar{E}_2 \cap E_1) = P(\bar{E}_1) \times P(\bar{E}_2) \times P(E_1) = \left(\frac{5}{6}\right)^2 \times \frac{1}{6}$$

Similarly, probability of A's winning in fifth throw is

$$p_2 = P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_1 \cap \bar{E}_2 \cap E_1)$$

$$= P(\bar{E}_1) \times P(\bar{E}_2) \times P(E_1) \times P(\bar{E}_2) \times P(E_1) = \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$$ and so on .

$\therefore$ A's chances of winning is : $P(A) = p_1 + p_2 + p_3 + ...$

[By addition theorem]

$$= \frac{1}{6} + \left(\frac{5}{6}\right)^2 + \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + ... = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{\frac{1}{6}}{\frac{11}{36}} = \frac{6}{11}$$

Similarly B's chances of winning

$$= \frac{5}{6} \times \frac{1}{6} + \left(\frac{5}{6}\right)^3 + \frac{1}{6} + \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + ... = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{\frac{5}{36}}{\frac{11}{36}} = \frac{5}{11}.$$

Therefore for a prize of Rs. 99 :

A's expectation = $\frac{6}{11} \times \text{Rs. } 11 = \text{Rs.} 6$

and B's expectation = $\frac{5}{11} \times \text{Rs. } 11 = \text{Rs.} 5$

82.(D) Let

$(x_i - 22) = u_i$ and $y_i - 19 = v_i$ then

$\Sigma u_i = \Sigma x_i - 22 \times 10 = 5,$

$\Sigma v_i = 189 - 19 \times 10 = -2$

But $$r_{X,Y} = r_{U,V} = \frac{\frac{1}{n}\sum u_i v_i - \bar{u}\,\bar{v}}{\sqrt{\frac{1}{n}\sum u_i^2 - \bar{u}^2}\sqrt{\frac{1}{n}\sum v_i^2 - \bar{v}^2}}$$

$$= \frac{\frac{1}{10} \times 47 - \left(\frac{1}{10} \times 5\right)\left(\frac{1}{10} \times (-2)\right)}{\sqrt{\frac{1}{10} \times 9.25 - \frac{1}{4}}\sqrt{\frac{1}{10} \times 4.04 - \frac{1}{25}}}$$

$$= \frac{4.7 + 0.1}{\sqrt{92.5 - 0.25}\sqrt{40.4 - .04}} = \frac{4.8}{3 \times 2} = 0.8$$

83.(C) We have $b_{YX} = r\dfrac{\sigma_Y}{\sigma_X}$ and $b_{XY} = r\dfrac{\sigma_X}{\sigma_Y}$ so $b_{YX}\, b_{XY} = r^2$. Therefore

$r = -\sqrt{b_{XY} b_{YX}}$ as the given distribution is negatively correlated .

84.(A) Since G.M. > H.M. so

$$\sqrt{b_{YX} b_{XY}} > \frac{2b_{YX} b_{XY}}{b_{YX} + b_{XY}} \Rightarrow \frac{r}{2} > \frac{1}{\dfrac{1}{b_{XY}} + \dfrac{1}{b_{YX}}} \left(r > 0 \text{ as } b_{YX} > 0\right)$$

$$\Rightarrow \qquad \frac{2}{r} < \frac{1}{b_{XY}} + \frac{1}{b_{YX}}$$

85.(A) We have :

$$\overline{X} = \frac{\Sigma x}{N} = \frac{120}{12} = 10 \; ; \; \overline{Y} = \frac{\Sigma Y}{N} = \frac{432}{12} = 36$$

b_{yx} = Coefficient of regression of Y on X

$$= \frac{\Sigma XY - \dfrac{(\Sigma X)(\Sigma X)}{N}}{\left\{\Sigma X^2 - \dfrac{(\Sigma X)^2}{N}\right\}} = \frac{4992 - \dfrac{120 \times 432}{12}}{\left\{1392 - \dfrac{(120)^2}{12}\right\}} = \frac{4992 - 4320}{1392 - 1200} = \frac{672}{192} = 3.5$$

b_{yx} = Coefficient of regression of Y on X

$$\frac{\Sigma XY - \dfrac{(\Sigma X)(\Sigma Y)}{N}}{\left\{\Sigma Y^2 - \dfrac{(\Sigma Y)^2}{N}\right\}} = \frac{4992 - \dfrac{120 \times 432}{12}}{\left\{18252 - \dfrac{(432)^2}{12}\right\}} = \frac{672}{2700} = 0.249.$$

86.(D) Regression equations of Y on X

$$Y - \overline{Y} = b_{YX}\left(X - \overline{X}\right)$$

or $\quad Y - 36 = 3.5(X - 10)$

$$Y = 3.5X + 1$$

87.(C) The sample space is given by

S = {11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66}

We have A = {21, 22, 23, 24, 25, 26, 41, 42, 43, 44, 45, 46, 61, 62, 63, 64, 65, 66}

B = {11, 13, 15, 21, 23, 25, 31, 33, 35, 41, 43, 45, 51, 53, 55, 61, 63, 65}

Note that $A \cap B = \phi$ s, A and B cannot be mutually exclusive.

Next, $P(A) = \frac{18}{36} = \frac{1}{2}, P(B) = \frac{18}{36} = \frac{1}{2}$

$P(A \cap B) = \frac{9}{36} = \frac{1}{4}, = P(A)P(B)$

A and B are independent.

88.(A) We are given P (A) = 0.25. P (B) = 0.50, $P(A \cap B)$ = 0.14

Now,

P (neither A nor B)

= $P(A' \cap B') = 1 - P(A \cap B)$

= 1- [P (A) + P(B) - s$(A \cap B)$]

= 1 - [0.25 + 0.50 - 0.14]

= 1 - 0.61 = 0.39.

89.(C)

Let A, B and C be the events that the student is successful in tests I, II, III respectively. Then

P (the student is successful)

= $P[A \cap B \cap C'] \cup (A \cap B' \cap C) \cup (A \cap B \cap C)$

= $P[A \cap B \cap C'] + P(A \cap B' \cap C) + P(A \cap B \cap C)$

= $P(A)P(B)P(C') + P(A)P(B')P(C) + P(A)P(B)P(C)$

[$\because$ A, B and C are independent]

= p q (1 - 1/2) + p (1 - q) (1/2) + (pq) (1/2)

$= \frac{1}{2} = \frac{1}{2}p(1+q) \quad \Rightarrow \quad p(1+q) = 1$

90.(C) Put b = $\frac{da}{dt}$ and c = $\frac{d^2a}{dt^2}$ in the result of (i) above

Then $\frac{d}{dt}\left[a \frac{da}{dt} \frac{d^2a}{dt^2}\right] = \left[\frac{da}{dt} \frac{da}{dt} \frac{d^2a}{dt^2}\right] + \left[a \frac{d^2a}{dt^2} \frac{d^2a}{dt^2}\right] + \left[a \frac{da}{dt} \frac{d^3a}{dt^3}\right]$

$\because \frac{db}{dt} = \frac{d^2a}{dt^2}$ and $\frac{dc}{dt} = \frac{d}{dt}\left(\frac{d^2a}{dt^2}\right) = \frac{d^3a}{dt^3}$

$= \left[a \frac{da}{dt} \frac{d^3a}{dt^3}\right] \qquad \because [a\ a\ b] = 0$

91.(D) $\nabla f = \left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial y}\right)(x^2 y + y^2 x + z^2)$

$= \mathbf{i}\frac{\partial}{\partial x}(x^2 y + y^2 x + z^2) + \mathbf{j}\frac{\partial}{\partial y}(x^2 y + y^2 x + z^2) + \mathbf{k}\frac{\partial}{\partial y}(x^2 y + y^2 x + z^2)$

$= \mathbf{i}(2xy + y^2) + \mathbf{j}(x^2 + 2yx) + \mathbf{k}(2z)$

At (1, 1, 1) we have

$\nabla f = \mathbf{i}(2.1.1 + 1^2) + \mathbf{j}(1^2 + 2.1.1) + \mathbf{k}(2.1) = 3i + 3j + 2k$

92.(C) $|r| = \sqrt{(x^2 + y^2 + z^2)}$ or $|r|^3 = (x^2 + y^2 + z^2)^{3/2}$

$\nabla |r|^3 = \left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial y}\right)(x^2 + y^2 + z^2)^{3/2}$

$= \mathbf{i}\frac{\partial}{\partial x}(x^2 + y^2 + z^2)^{3/2} + \mathbf{j}\frac{\partial}{\partial y}(x^2 + y^2 + z^2)^{3/2} + \mathbf{k}\frac{\partial}{\partial y}(x^2 + y^2 + z^2)^{3/2}$

$= \mathbf{i}\left[\frac{3}{2}(x^2 + y^2 + z^2)^{1/2} . 2x\right] + \mathbf{j}\left[\frac{3}{2}(x^2 + y^2 + z^2)^{1/2} . 2y\right] + \mathbf{k}\left[\frac{3}{2}(x^2 + y^2 + z^2)^{1/2} . 2z\right]$

$= 3(x^2 + y^2 + z^2)^{1/2}[x\mathbf{i} + y\mathbf{j} + z\mathbf{k}] = 3r\,\mathbf{r}$

93.(B) We know r = x**i** + y**j** + z**k** or r = | r | = $\sqrt{(x^2+y^2+z^2)}$

f = $r^2 e^{-r}$, as $r^2 = x^2 + y^2 + z^2$

$2r\frac{\partial r}{\partial x} = 2x$ or $\frac{\partial r}{\partial x} = \frac{x}{r}$ .... (i)

Similarly $\frac{\partial r}{\partial y} = \frac{y}{r}$, $\frac{\partial r}{\partial z} = \frac{z}{r}$

$\nabla f = \left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right) r^2 e^{-r}$

$= \sum i \frac{\partial}{\partial x}(r^2 e^{-r}) = \sum i\ [(2 r e^{-r} - r^2 e^{-r} r)]\frac{\partial r}{\partial x}$

$= \sum i\ [(2-r)\, r\, e^{-r}\frac{x}{r}]$, from (i) $= (2-r)e^{-r}\sum xi = (2-r)\, e^{-r}\,\mathbf{r}$.

94.(A) $\nabla e^{r^2} = \left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right)e^{r^2} = \mathbf{i}\ 2\ r\ e^{r^2}\frac{\partial r}{\partial x} + \mathbf{j}\ 2\ r e^{r^2}\frac{\partial r}{\partial y} + \mathbf{k}\ 2\ r e^{r^2}\frac{\partial r}{\partial z} = 2\ r$

$e^{r^2}\left[\frac{\partial r}{\partial x}\mathbf{i}+\frac{\partial r}{\partial y}\mathbf{j}+\frac{\partial r}{\partial z}\mathbf{k}\right] = 2\ r\ e^{r^2}\left[\frac{x}{r}\mathbf{i}+\frac{y}{r}\mathbf{j}+\frac{z}{r}\mathbf{k}\right]$

$\because r^2 = x^2 + y^2 + z^2 \Rightarrow 2r\frac{\partial r}{\partial x} = 2x$ or $\frac{\partial r}{\partial x} = \frac{x}{r}$ etc. $= 2e^{r^2}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = 2e^{r^2}\mathbf{r}$.

95.(A) Let r = $|\vec{r}|$ = |x**i** + y**j** + z**k**|

Then log $|\vec{r}|$ = log r , where $r^2 = x^2 + y^2 + z^2$

grad {log $|\vec{r}|$ } = grad (log r) = $\left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right)$ = log r $= \sum\left[\mathbf{i}\frac{\partial}{\partial x}(\log r)\right] = \sum\mathbf{i}\left[\frac{1}{r}\frac{\partial r}{\partial x}\right] =$

$\sum\mathbf{i}\left[\frac{1}{r}.\frac{x}{r}\right] . \frac{\partial r}{\partial x} = \frac{x}{r} = \frac{1}{r^2}\sum x\mathbf{i} = \frac{1}{r^2}\vec{r} = \frac{\vec{r}}{|\vec{r}|^2}$

96.(C) $\nabla r^{-3} = \left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right) r^{-3} = \sum \mathbf{i}\frac{\partial}{\partial x}(r^{-3})$

$= \sum\mathbf{i}\left(-3r^{-4}\frac{\partial r}{\partial x}\right)$, where $r^2 = x^2 + y^2 + z^2$ $= \sum\mathbf{i}\left(-3r^{-4}\frac{x}{r}\right)$

Q $\frac{\partial r}{\partial x} = \frac{x}{r}$ $= -3r^{-5}\sum x\mathbf{i} = -3r^{-5}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = -3r^{-5}\,\mathbf{r}$

97.(C) grad f = $\nabla f = \nabla \log\sqrt{(x^2+y^2)} = 2\nabla \log\sqrt{(x^2+y^2)} = \frac{1}{2}\left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right)\log(x^2+y^2)$

$= \frac{1}{2}\left[\mathbf{i}\frac{\partial}{\partial x}\log(x^2+y^2)+\mathbf{j}\frac{\partial}{\partial y}\log(x^2+y^2)+\mathbf{k}\frac{\partial}{\partial z}\log(x^2+y^2)\right] = \frac{1}{2}\left[\left\{\frac{1}{x^2+y^2}2x\right\}\mathbf{i}+\left\{\frac{1}{x^2+y^2}2y\right\}\mathbf{j}\{0\}\mathbf{k}\right]$

$= (x\mathbf{i} + y\mathbf{j}) / (x^2 + y^2) = \frac{(x\mathbf{i}+y\mathbf{j})}{(x\mathbf{i}+y\mathbf{j}).(x\mathbf{i}+y\mathbf{j})}$, $\quad$ i . **i** = 1, i . **j** = 0

$= \frac{r - z\mathbf{k}}{(r-z\mathbf{k}).(r-z\mathbf{k})}$ $\quad$ r = x**i** + y**j** + z**k**

$= \frac{r-(k.r)\mathbf{k}}{\{r-(k.r)\mathbf{k}\}.\{r-(k.r)\mathbf{k}\}}$ $\quad$ **k . r = k** . {x**i** + y**j** + z**k**} = z

98.(A) Let **a** = a_1 **i** + a_2 **j** + a_3 **k**

a. $\nabla = (a_1\,\mathbf{i} + a_2\,\mathbf{j} + a_3\,\mathbf{k})\,.\left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right) = a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z}$

a. $\nabla r = \left(a_1\frac{\partial}{\partial x}+a_2\frac{\partial}{\partial y}+a_3\frac{\partial}{\partial z}\right)$ (x**i** + y**j** + z**k**) $\quad$ r = x**i** + y**j** + z**k**

= a_1 **i** + a_2 **j** + a_3 **k** = a $\quad$ Hence Proved

99.(A) ∇**.** F = $\frac{\partial F_1}{\partial x}+\frac{\partial F_2}{\partial y}+\frac{\partial F_3}{\partial z}$, where F = F_1 **i** + F_1 **j** + F_3 **k** .... (i)

Now F = ∇ $(x^3 + y^3 + z^3 - 3xyz)$

$= \mathbf{i}\frac{\partial}{\partial x}(x^3 + y^3 + z^3 - 3xyz) + \mathbf{j}\frac{\partial}{\partial y}(x^3 + y^3 + z^3 - 3xyz) + \mathbf{k}\frac{\partial}{\partial z}(x^3 + y^3 + z^3 - 3xyz)$

= **i**$(3x^2 - 3yz)$ + **j** $(3y^2 - 3xz)$ + **k** $(3z^2 - 3xy)$

From (i) we have

∇**.** F = $\frac{\partial}{\partial x}(3x^2 - 3yz) + \frac{\partial}{\partial y}(3y^2 - 3xz) + \frac{\partial}{\partial z}(3z^2 - 3xy)$

= 6x + 6y + 6z = 6 (x + y + z)

100.(B) div **r** = div (x **i** + y **j** + z **k**)

$= \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z)$ $\quad$ = 1 + 1 + 1 = 3 $\quad$ Hence proved

101.(B) div F = $\frac{\partial}{\partial x}(xy^2) + \frac{\partial}{\partial y}(2x^2 yz) + \frac{\partial}{\partial z}(-3y z^2) = y^2 + 2x^2 z - 6yz$

div F at (1, –1, 1) = $(-1)^2 + 2(1)^2 . 1 - 6(-1) . 1 = 9$

102.(A) div $\left(\frac{\mathbf{r}}{r}\right) = \nabla.\left(\frac{\mathbf{r}}{r}\right)$

$$= \left(\mathbf{i}\frac{\partial}{\partial x}+\mathbf{j}\frac{\partial}{\partial y}+\mathbf{k}\frac{\partial}{\partial z}\right) . \left(\frac{x}{r}\mathbf{i}+\frac{y}{r}\mathbf{j}+\frac{z}{r}\mathbf{k}\right) = \frac{\partial}{\partial x}\left(\frac{x}{r}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r}\right)$$

$$= \left[\frac{1}{r}-\frac{x}{r^2}\frac{\partial r}{\partial x}\right] + \left[\frac{1}{r}-\frac{y}{r^2}\frac{\partial r}{\partial y}\right] + \left[\frac{1}{r}-\frac{z}{r^2}\frac{\partial r}{\partial z}\right] \text{ or div } \left(\frac{\mathbf{r}}{r}\right) = \frac{3}{r}-\frac{1}{r^2}\left[x\frac{\partial r}{\partial x}+y\frac{\partial r}{\partial y}+\frac{\partial r}{\partial z}\right], \text{ where } \frac{\partial r}{\partial x} = \frac{x}{r} \text{ etc.}$$

$$= \frac{3}{r}-\frac{1}{r^2}\left[x\left(\frac{x}{r}\right)+y\left(\frac{y}{r}\right)+z\left(\frac{z}{r}\right)\right] = \frac{3}{r}-\frac{1}{r^3}[x^2+y^2+z^2] = \frac{3}{r}-\frac{1}{r^3}(r^2)\; r^2 = x^2+y^2+z^2$$

$$= \frac{3}{r}-\frac{1}{r} = \frac{2}{r}$$

103.(A) Total work done = $\int_C \mathbf{F}.d\mathbf{r}$

$$= \int_C [3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}] . (\mathbf{i}dx + \mathbf{j}dy + \mathbf{k}dz)] = \int_C [3xy\, dx - 5z\, dy + 10\, x\, d\, z]$$

$$= \int_{t=0}^{2} [3t\,(t^2+1)\, d\,(t) - 5t^3\, d\,(t^2+1) + 10\, td\,(t)^3]$$

Putting $x = t$, $y = t^2 + 1$, $z = t^3$

$$= \int_0^2 3(t^3+t)\, dt - 5\int_0^2 2t^4\, dt + 10\int_0^2 3t^3\, dt = 3\left[\frac{1}{4}t^4+\frac{1}{4}t^2\right]_0^2 - 5\left[\frac{2}{5}t^5\right]_0^2 + \frac{15}{2}\left[t^4\right]_0^2$$

$$= 3\,[4+2] - 5\left[\frac{64}{5}\right]+\frac{15}{2}[16] = 18 - 64 + 120 = 74$$

104.(D) By Gauss divergence theorem, we have $\int_S \mathbf{F.n}\, dS = \int_V \text{div } \mathbf{F} dV =$

$\int_V \nabla.\mathbf{F}\, dV = \int_V \nabla.(\nabla\phi)dV,$

∵ $\mathbf{F} = \nabla\phi$ = $\int_V \nabla^2\phi\, dV = \int_V (-4\pi\rho)dV,$

∵ $\nabla^2\phi = -4\pi\rho$ (given) = $-4\pi\int_V \rho\, dV$

105.(B) By Gauss divergence theorem, we have

Put **F** = ϕA, where A is an arbitrary constant non-zero vector.

Then $\int_S \phi \mathbf{A}.\mathbf{n}\, dS = \int_V \text{div}\,(\phi A) dV$ or $\mathbf{A}. \int \phi\, \mathbf{n}\, dS = \int_V [\phi\, \text{div}\, \mathbf{A} + (\nabla\phi).\mathbf{A}] dV$, = $\int_V (\nabla\phi).\, \mathbf{A}\, dV$,

div **A** = 0 as **A** is constant vector.

or $\mathbf{A}\,.\, \int_S \phi\, \mathbf{n}\, dS = \mathbf{A}\,.\, \int_V (\nabla\phi) dV$

A is constant vector, it can be taken outside the sign of integration

or $\mathbf{A}\,.\left[\int_S \phi\, \mathbf{n}\, dS - \int_V (\nabla\phi) dV\right] = 0$ or $\int_S \phi\, \mathbf{n}\, dS - \int_V \nabla\phi\, dV$, **A** is an arbitrary vector.

106.(C) Putting **F** = f**A** in Gauss' theorem we have

$$\int_S \phi\, \mathbf{A.n}\, dS = \int_V \text{div}\,(\phi\, A) dS = \int_V [\phi\, \text{div}\, \mathbf{A} + \nabla\phi.\mathbf{A}] dV = \int_V \phi\, \text{div}\, \mathbf{A}\, dV + \int_V \mathbf{A}.\nabla\,\phi\, dV$$

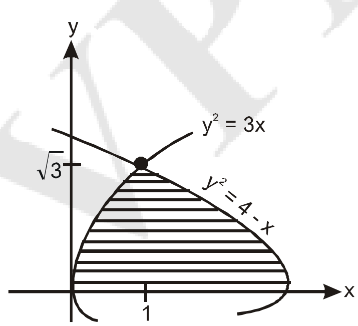
or $\int_V \mathbf{A}.\nabla\phi\, dV = \int_S \phi\, \mathbf{A.n}\, dS - \int_V \phi\, \text{div}\, \mathbf{A} dV$

107.(C) It is convenient to evaluate I by means of strips parallel to the x-axis.

$$I = \int_0^{\sqrt{3}} \int_{y^2/3}^{4-y^2} (x-y) dx\, dy = \int_0^{\sqrt{3}} \left(\frac{1}{2}x^2 - yx\right)\Bigg]_{y^2/3}^{4-y^2} dy = \int_0^{\sqrt{3}} \left[\frac{1}{2}\left(4-y^2\right)^2 - y\left(4-y^2\right)\right] - \left[\frac{1}{2}\left(y^2/3\right)^2 - y^3/3\right] dy$$

$$= \int_0^{\sqrt{3}} \left(8 - 4y^2 + \frac{1}{2}y^4 - 4y + y^3 - \frac{1}{18}y^4 - \frac{1}{3}y^3\right) dy = \int_0^{\sqrt{3}} \left(8 - 4y - 4y^2 + \frac{2}{3}y^3 + \frac{4}{9}y^4\right) dy$$

$$= \left[8y - 2y^2 - \frac{4}{3}y^3 + \frac{1}{6}y^4 + \frac{1}{6}y^4 + \frac{4}{45}y^5\right]_0^{\sqrt{3}} = 8\sqrt{3} - 6 - 4\sqrt{3} + \frac{3}{2} + \frac{4}{5}\sqrt{3} = \frac{24}{5}\sqrt{3} - \frac{9}{2}.$$



115.(D) We are given $P(E\cap F)=1|2$ and $P(E'\cap F')=1/2$

As E and F are independent, we get P(E) P (F) = 1 | 2 and P(E') P (F') = 1/2

$\Rightarrow$ (1 - P (E)) + (1 - P (F)) = 1/2

$\Rightarrow$ (1 - (P (E) + P (F)) + P (E) P (F) = 1/2

$\Rightarrow$ P (E) + P (F) = 1+ 1/12 - 1/2 = 7/12

$\therefore$ Equations whose roots are P (E) and P (F) is

x^2 - (P (E) + P (F)) x + P (E) P (F) = 0

or $x^2-\frac{7}{12}x+\frac{1}{12}$ $\Rightarrow$ $12x^2 - 7x + 1 = 0$

$\Rightarrow$ (3x - 1) (4x - 1) = 0 $\Rightarrow$ $x=\frac{1}{3},\frac{1}{4}$,

P (E) < P (F), we take P (E) = $\frac{1}{4}$ and P (F) = $\frac{1}{3}$

116.(A) Let E denote the event that a six occurs and A the event that the man reports that it is a six. We have P (E) = 1/6, P (E') = 5/6, P (A | E) = 3/4 and P(A | E') = 1/4. By Baye's theorem

$$P(E\mid A) = \frac{P(E)\,P(A\mid E)}{P(E)\,P(A\mid E)+P(E')\,P(A\mid E')} = \frac{(1/6)(3/4)}{(1/6)(3/4)+(5/6)(1/4)}=\frac{3}{8}.$$

117.(A) Since (1 + 3p)/3, (1 - p) / 4 and (1 - 2p) / 2 are the probabilites of the three events we must have

$0\le\frac{1+3p}{3}\le1,\quad 0\le\frac{1-p}{4}\le1$ and $0\le\frac{1-2p}{2}\le1$

$\Rightarrow$ $0\le1+3p\le3,\quad 0\le1-p\le4,$ and $0\le1-2p\le2$

$\Rightarrow$ $-1\le3p\le2,\quad -3\le p\le1,$ and $-1\le2p\le1$

$\Rightarrow$ $-\frac{1}{3}\le p\le\frac{2}{3},\quad -3\le p\le1,$ and $-\frac{1}{2}\le p\le\frac{1}{2}$

Also, as (1 + 3p) / 3, (1 -p)/4 and (1 - 2p)/2 are the probabilites of three mutually exclusive events

$$0\le\frac{1+3p}{3}+\frac{1-p}{4}+\frac{1-2p}{2}\le1$$

$\Rightarrow$ $0\le4+12p+3-3p+6-12p\le12$

$\Rightarrow \quad 0 \le 13 - 3p \le 12 \quad \Rightarrow 1 \le 3p \le 13$

$\Rightarrow \quad 1/3 \le p \le 13/3$

Thus, the required values of p are such that

$$\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le p \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\}$$

$\Rightarrow \quad 1/3 \le p \le 1/2.$

119.(C) discriminant D of the quadratic equation

$$x^2 + mx + \frac{1}{2} + \frac{m}{2} = 0$$

is given by

$$D = m^2 - 4\left(\frac{1}{2} + \frac{m}{2}\right) = m^2 - 2m - 2 = (m - 1)^2 - 3$$

Now, $D \ge 0 \quad \Leftrightarrow \quad (m - 1)^2 \ge 3$

This possible for m = 3, 4 and 5. Also the total no of ways of choosing m is 5.

$\therefore$ probability of the required event = 3/5.

120.(B) The probability that one two tests needed = (probability that the first machines tested is faulty) × (probability that the second machine tested is faulty given that the first machine is faulty) $= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$.